SAHC

ADVANCED MASTERS IN STRUCTURAL ANALYSIS
OF MONUMENTS AND HISTORICAL CONSTRUCTIONS

Master's Thesis

Omar Alberto Pino Acuña

Numerical Analysis of the Strength Capacity of Timbrel and Composite Timbrel-Concrete Vaults

UNIVERSITAT POLITÈCNICA DE CATALUNYA

University of Minho

Spain | 2019

Numerical Analysis of the Strength Capacity of Timbrel and Composite Timbrel-Concrete Vaults

Omar Alberto Pino Acuña

Spain | 2019

ADVANCED MASTERS IN STRUCTURAL ANALYSIS
OF MONUMENTS AND HISTORICAL CONSTRUCTIONS

Master's Thesis

Omar Alberto Pino Acuña

Numerical Analysis of the Strength Capacity of Timbrel and Composite Timbrel-Concrete Vaults

Spain | 2019

DECLARATION

Name:	Omar Alberto Pino Acuña
Email:	opino31@gmail.com

Title of the Msc Dissertation:	Numerical analysis of the strength capacity of timbrel and composite timbrel-concrete vaults
Supervisor(s):	Pere Roca Fabregat
Year:	2019

I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

I hereby declare that the MSc Consortium responsible for the Advanced Masters in Structural Analysis of Monuments and Historical Constructions is allowed to store and make available electronically the present MSc Dissertation.

University:	Technical University of Catalonia
Date:	19 – 07 - 2019
Signature:	

This page is left blank on purpose.

This work is dedicated to my family, for giving me all their support.

This page is left blank on purpose.

ACKNOWLEDGEMENTS

First and foremost, I would like to thank to the SAHC program for the scholarship and for the possibility of studying the structural analysis of historical constructions. Besides I would like to thank to all my professors, staff members and my classmates from the Universities of the Consortium, for their knowledge and compromise with the students, especially to my supervisors Prof. Ing Pere Roca Fabregat and my co- tutor Daniel Lopez Lopez for the guidance and expertise during the development of this thesis.

Finally, but not least I would like to thank to my wife Carla Sánchez Santana and my son Maximiliano Pino Sánchez for the great effort they made in accompanying me to study in Europe, I will never finish giving back so much support

This page is left blank on purpose.

ABSTRACT

The timbrel vault is a structural masonry system composed by thin bricks, fast – setting cement and mortar, built by placing the bricks flatly in courses, the first course acts as a formwork of the following layers, so no additional formwork is needed, this is one of the most important characteristic of them, because allows economizing formwork material, which added to the high fire resistance and aesthetic value transform it into an attractive structural system. Taking into consideration that the tile vault can act as a formwork, concrete can be poured above them and steel reinforcement can be also placed without the need of formwork.

Through several authors we know nowadays that this system could have originated in Rome or Byzantium and it spread along the coast of the Mediterranean Sea arriving in Spain and more specifically to Catalonia from where they are traditionally known and used during centuries even exporting the system to other countries as did Rafael Guastavino and his son who built a lot of timbrel vault in the United States.

The apparition of other constructive system like the reinforced concrete and steel, more industrialized methods added to the development of the design codes, ended with the use of the timbrel vaults.

Nevertheless, in the XXI century the timbrel vault system has regained interest due to the sustainability of the system construction and their aesthetic value. Recently investigations have used modern techniques of structural analysis like Finite Elements Methods and experimental testing. Between them, David Lopez developed an experimental research of timbrel and composite timbrel – concrete vaults in the Technical University of Catalonia.

From the experimental test, numerical analyses through a non-linear macro model with Finite Elements Models were developed in order to reproduce in the best possible way the behavior of the tested prototypes. From these results a comparison and analysis of results is performed.

Keywords: timbrel vault, Composite vault, non-linear analysis, finite element method, masonry, historical structures, structural analysis.

This page is left blank on purpose.

RESUM

La volta de maó de pla és un sistema de maçoneria estructural compost per maons prims, morter i ciment d'enduriment ràpid, es construeix col·locant els maons de manera plana, la primera filera actua com un encofrat de les següents, de manera que no es necessita un encofrat addicional . És una de les seves característiques més importants, ja que permet economitzar material d'encofrat, el que sumat a l'alta resistència al foc i al valor estètic les transforma en un atractiu sistema estructural. Tenint en compte que la volta de maó de pla pot actuar com un encofrat, el formigó es pot abocar per sobre d'ells i l'armadura també es pot col·locar sense necessitat d'encofrats.

A través de diversos autors, avui dia sabem que aquest sistema podria haver-se originat a Roma o Bizanci i que posteriorment es va estendre al llarg de la costa del Mar Mediterrani arribant a Espanya i més específicament a Catalunya, des d’un es coneixen i utilitzen tradicionalment durant segles, fins i tot exportant el sistema a altres països. com ho van fer Rafael Guastavino i el seu fill, qui van construir moltes voltes tapiades als Estats Units.

L'aparició d'altres sistemes constructius com el formigó armat i l'acer, mètodes més industrialitzats sumats al desenvolupament dels codis de disseny, va acabar amb l'ús de les voltes de maó de pla.

No obstant això, al segle XXI, el sistema de volta de maó de pla ha recuperat l’ interès a causa de la sostenibilitat del sistema constructiu i el seu valor estètic. Investigacions recents han utilitzat tècniques modernes d'anàlisi estructural com mètodes d'elements finits i proves experimentals. Entre ells, David López va desenvolupar una investigació experimental de voltes de maó de pla i compostes la Universitat Tècnica de Catalunya.

A partir d'aquests assajos experimentals, es van desenvolupar una sèrie anàlisi numèrics a través d'un macro model no lineal amb elements finits per reproduir de la millor manera possible el comportament dels prototips provats. A partir d'aquests resultats es realitza una comparació i anàlisi dels resultats.

Paraules clau: Volta de maó de pla, volta composta, anàlisi no lineal, mètode elements finits, maçoneria, construccions històriques, anàlisi estructural.

This page is left blank on purpose.

RESUMEN

La bóveda tabicada es un sistema de mampostería estructural compuesto por ladrillos delgados, mortero y cemento de fraguado rápido, se construye colocando los ladrillos de manera plana, la primera hilera actúa como un encofrado de las siguientes, por lo que no se necesita un encofrado adicional. Es una de sus características más importantes, ya que permite economizar material de encofrado, lo que sumado a la alta resistencia al fuego y al valor estético las transforma en un atractivo sistema estructural. Teniendo en cuenta que la bóveda tabicada puede actuar como un encofrado, el hormigón se puede verter por encima de ellos y la armadura también se puede colocar sin necesidad de encofrados.

A través de varios autores, hoy en día sabemos que este sistema podría haberse originado en Roma o Bizancio y que posteriormente se extendió a lo largo de la costa del Mar Mediterráneo llegando a España y más específicamente a Cataluña, desde donde se conocen y utilizan tradicionalmente durante siglos, incluso exportando el sistema a otros países. como lo hicieron Rafael Guastavino y su hijo, quienes construyeron muchas bóvedas tabicadas en los Estados Unidos.

La aparición de otros sistemas constructivos como el hormigón armado y el acero, métodos más industrializados sumados al desarrollo de los códigos de diseño, terminó con el uso de las bóvedas tabicadas.

Sin embargo, en el siglo XXI, el sistema de bóveda de tabicada ha recuperado el interés debido a la sustentabilidad del sistema constructivo y su valor estético. Investigaciones recientes han utilizado técnicas modernas de análisis estructural como métodos de elementos finitos y pruebas experimentales. Entre ellos, David López desarrolló una investigación experimental de bóvedas tabicadas y compuestas la Universidad Técnica de Cataluña.

A partir de esos ensayos experimentales, se desarrollaron una serie análisis numéricos a través de un macro modelo no lineal con elementos finitos para reproducir de la mejor manera posible el comportamiento de los prototipos probados. A partir de estos resultados se realiza una comparación y análisis de los resultados.

Palabras clave: Bóveda tabicada, bóveda compuesta, análisis no lineal, método elementos finitos, mampostería, construcciones históricas, análisis estructural.

This page is left blank on purpose.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

1. INTRODUCTION

In General, arches and vaults are structures used mainly to connect the span between two supports, also they can support weight above them, are mainly used in bridges and opening in walls like doors and windows. When continuous arches are supported in walls with the purpose of cover a roof can be designated as vault.

According to Jaques Heyman [1] "A masonry structure is an assemblage of stones – or bricks, or sun-dried clay - classified with certain distinct labels, as Byzantine, Romanesque, Gothic or Adobe, but recognized by engineers as having a common structural action". This assembly of blocks can be done with or without mortar, but considering the mortar has only the purpose of filling the voids and doesn't contribute to the resistance in compression. In general, masonry arches only resist compressive forces, however small tensile stresses can be accepted.

There are many ways to cover the roof of one building by using vaults, across history different structures have been developed to this purpose, the following it is a brief description of the main vault systems.

A barrel vault can be described as a series of parallel arcs placed side by side creating a continuous surface of semicircular shape, so the structural behaviour of these vaults is similar of the arches. According to Heyman [1] the thickness of these vaults should be the 10% of the radius of the semicircular section, so that means that a high amount of material will needed to build the vault, which added to the material used for formwork make it an inexpensive solution, considering also the thrusts forces due the self weight are greater and therefore more robust structures are needed to resist them.

The groined vault presents an improvement in relation to the barrel vault in terms of use of materials, also allows a different configuration of the supports, concentrating the load at the corners, due the thrust load is transferred by the edges of the intersection of the two perpendicular barrel vaults (groin).

The main problem with this system is the construction of the edges or groins of the vault because the stone block located in this point needs precise geometrical cuts to fit with the rest of the stones, the geometry of these stones is even more complicated when the perpendicular barrel vaults has different spans. Considering that, the builders of groined vaults in the Romanesque style began to build their vaults from those elements, named ribs. The builders discover that by building first the ribs as a framework, the geometrical problem the groined vault was solved, this vault was named ribbed vault.

Regarding with the domes, the main characteristic is that any point of the dome present two curvatures and any force acting in the dome will generate forces along this two curvatures. These curvatures can be defined as meridians (vertical direction) transferring the compression loads and parallels (horizontal direction) who transfer compression and tension loads, the distribution of this loads is analyzed from the top to the bottom of the dome, at the beginning the forces are in compression but the transform into tension forces approximately when the angle between the vertical axis and the cross section dome shape has a value of 50º, this tensile forces are called hoop forces.

The Timbrel or Catalan vaults are a constructive system built from bricks of small thickness and therefore low self weight. Their constructive system has the particularity that does not need any type of support or formwork. Considering the advantage of the non-use of formwork, it is possible also to add a layer of concrete above the Timbrel vault using it as a formwork, this is called Composite timbrel – concrete vault. The composite system significantly increases its resistance after the curing of the concrete; therefore it is more efficient from the point of view of the economy of construction materials and structural performance.

The timbrel vault system has been used since centuries in Europe but it’s typical in Catalonia where builders, architects and engineers use them, one of the most famous was Rafael Guastavino Sr. and their son who developed this system in the United States since the ends of the XIX century and the beginnings if the XX century. This system was putted aside in benefit of other structural system like steel and reinforced concrete during the middle of the XX century. Despite this its advantages remain and still a subject of study with the purpose of the better understanding of their behavior and developing of new analysis, construction and assessment techniques.

In order to know the behavior of this structural vault system, laboratory tests were carried out by David Lopez in a series of prototypes of timbrel vaults and composite timbrel - concrete vaults with simple and double curvatures. The tests were carried out taking the prototypes to the failure by applying a concentrated load at a certain point. From those tests, load - displacement, ultimate load and failure mechanism diagrams were obtained.

From the laboratory tests, a structural analysis will be developed by using a non-linear macro models with Finite Elements Models with the purpose of perform numerical simulations of the prototypes tested in order to calibrate the response obtained by simulating the geometry, supports conditions, load application material and their mechanical properties among others.

2. OBJECTIVES

The main purpose of this investigation is to develop a numerical simulation of the experimental campaign carried out by David Lopez [24] in the UPC facilities by developing a non – linear analysis by using a Macro - Modeling approach with Finite Elements Method. Several models were made for the different vaults tested in the software DIANA FEA version 10.3

Besides, for a better understanding of the timbrel vault system, a review of the history of these system and the authors who worked and reported their discoveries is also needed to be developed.

Considering the new methods used in the structural analysis nowadays, a review of how the timbrel and composite timbrel – concrete vaults are analyzed and assess a review of the state of knowledge regarding with this topic.

3. TIMBREL VAULTS

Timbrel vaults had been built since centuries ago and many authors have researched about this. In this point, a brief compilation of authors is carried out with the purpose of recognizing the contribution of these to the knowledge of the timbrel vaults at the present.

3.1 General

In relation to the vaults, three traditional types are mainly built in Catalonia: The *volta d' escala* (Figure 1) which is a system mainly used in the construction of staircases, where a group of timbrel vaults made by thin bricks placed flatly are superposed until reaching the desired level, then the steps are built above the vault. This system can be considerate as a variation of the timbrel vault but has in general three supports: the short edges with two timbrel vaults that correspond to the same stair and in one of the long edges with the wall of the main building.

Figure 1 – Volta d' Escala vault system (Ceramica Baucells)

Other vault system is the R*evoltó* (Figure 2) which is used in the floors of the buildings, this system is characterized by the use of wooden beams mainly (iron or steel beams can also be used) which are supported on the walls of the building evenly spaced, between this beams barrel vaults are built and supported on them by placing the bricks flatly.

Figure 2 – Revoltó vault system (Ceramica Baucells)

Finally, the timbrel vaults, also called tile o Catalan vaults are masonry structures made with thin bricks (10 - 15 millimeters) placed flatly, fast setting cement or gypsum for the first layer and simple cement mortar or lime to be used in the following courses and between them. The small thickness of the brick is an important issue because implies in a small self weight which allows the construction without the need for formwork or any support, to achieve the stability of the vault, for the first course it is necessary to use fast-setting cement or gypsum, for the following courses you can use simple cement mortar or lime.

The resistance of timbrel vaults is very high due to the fact that the mortar used between courses and also between the bricks inside the course doesn't allow the sliding between the courses, maintaining their shape under applied loads. The capacity of these structures is reached very fast such the day after being built the builders can walk above them without any problem.

The capacity of the vaults to be built without formwork and their resistance allows that concrete can be poured over it, creating a second layer with higher capacity. From a constructive point, this composite system allows to economize time and material without by not use formwork.

3.2 Historical overview

According to the origins of the timbrel vaults, different authors have written about it, Felix Cardellach [2] indicates that the timbrel vaults origin is in the Caracalla Baths in Rome, and later spreads throughout the Latin coast from Naples to Cartagena in Spain, then was exported to the interior regions of Europe. Choisy in his book *L'art de batir chez les Romains* [3] also indicates their use in

Caracalla baths and also explain that these vaults consisted in a first course of ceramic bricks with a thickness between 40 – 50 mm and 600 mm of width placed flat and above them smaller bricks with length of 1/3 of the first layer bricks also placed flatly, others bricks were placed perpendicular as can be seen in Figure 3. Over this vault, roman concrete was pounded in order to create the first example of composite vaults. Choisy [4] also mentioned that the timbrel vault method is analogous to the construction of brick vaults without formwork used in Byzantium.

In Catalonia, according to Bassegoda [5], the first reported constructions with this system were in the king’s Martin chapel in 1407, the Santa Cruz hospital cloister in 1417 and in the Pedralbes monastery in 1420

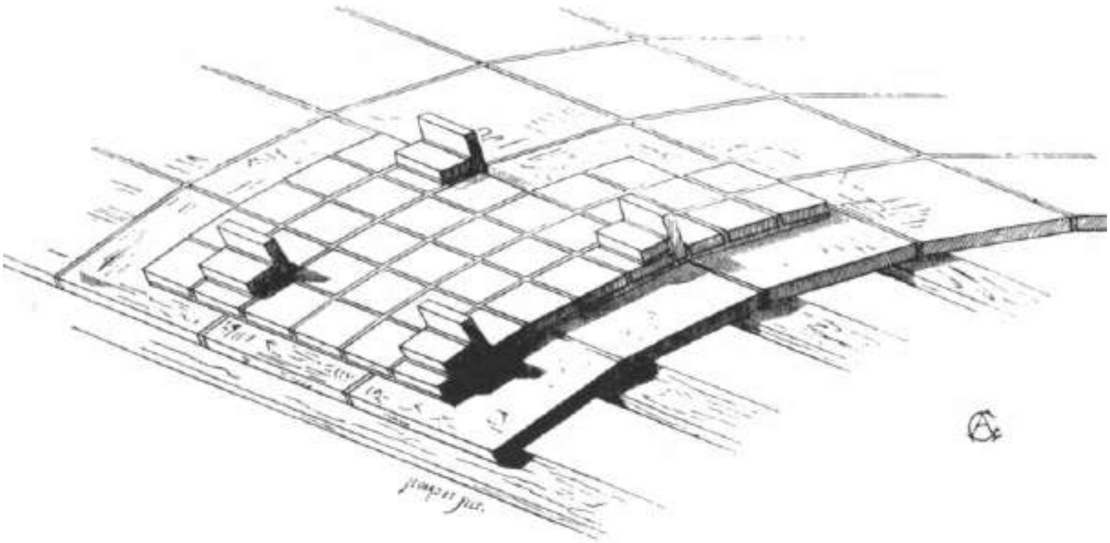


Figure 3 –Roman tile vault system (Choisy 1873)

According to Santiago Huerta [6], the most relevant text on construction and mechanics of timbrel vaults is the treatise of Fray Lorenzo de San Nicolás in 1639, he considered that perhaps the self weight of the timbrel vaults, lateral support must be applied to resist the thrust force, also indicates the need of fill the haunches for the first third of the vault ad transversal wall in the second third.

In France, the timbrel vaults system was adopted from Spain tradition. Comte d’ Espie in his book *“Manner of constructing all sorts of buildings from fire, or, a treatise upon the construction of arches made with bricks and plaster”,* called flat vaults. ” in 1754 he indicates the advantages of using this system, like the fire proof quality and also mentioned the differences regarding thrust rules with other vault systems, proposing a no thrust theory. Also in France, some test were described by Fontaine in his publication of 1865 titled *Experiences faites sur la stabilité des Voutes en briques* in this test, timbrel vaults were tested with geometry of 4 m x 6.25 m until failure that occurs at 1250 kg/m^2 and other one with 3.75 m span was loaded with 2700 kg/m^2 without failing. Based on Espie’s book, the

Spanish Benito Bails and Manuel Fornés developed new treatises, in the case of Fornés he sets out in great detail the way of building barrel vaults, staircases, domes, squinch arches, etc. [6].

This system was exported to the United States by the Valencian architect Rafael Gustavino, who settled in 1880. In Spain, he moved from Valencia to Barcelona to study architecture and in 1868 starts the construction of the Batlló Factory (Figure 4). Before he moves to the United States he won an award at the Philadelphia exhibition and once there he starts working as an architect and contractor. In 1889 creates his own Company, the Guastavino Fireproof Construction Company and tThe first contract was the Public Library of Boston (Figure 5). The vault system had a good reception in the United States due the audacity and economy in formwork.

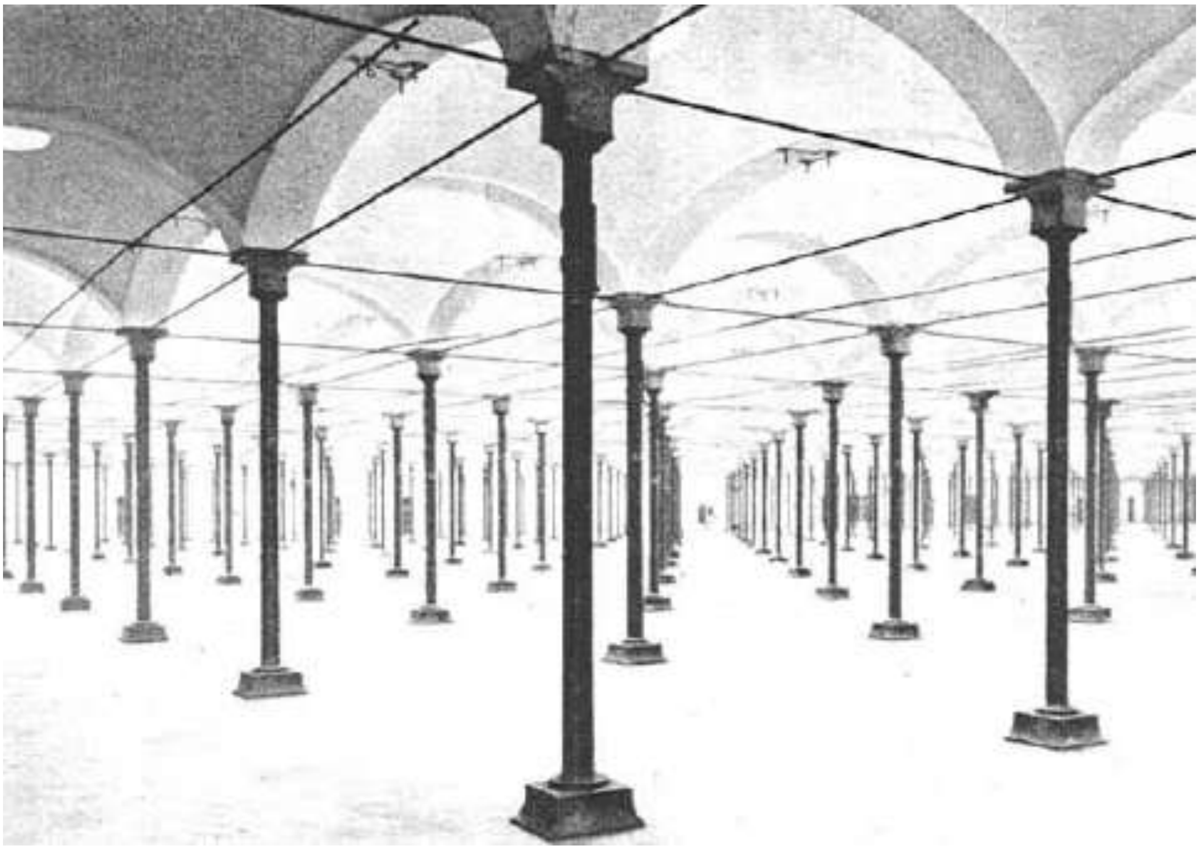


Figure 4 – Interior of Batllό Factory (Santiago Huerta, 2003)

In 1891 Guastavino registered a U.S. patent number 464,562 [7] where he presented their system in which the first course of bricks was built and after acted like a formwork for other layers of bricks or concrete, only a small wooden pieces needed to be used for obtain the geometry (Figure 6).

Figure 5 - Guastavino supervising the works in Boston Public Library (Santiago Huerta 2003)

In order to have a theoretical background of their system in 1892 published the book *Essay on the Theory and History of Cohesive Constructions applied especially to the timbrel arch*. In this publication Guastavino classified the arches and vaults in mechanical and cohesive and explained the advantages of the cohesive over the mechanical.

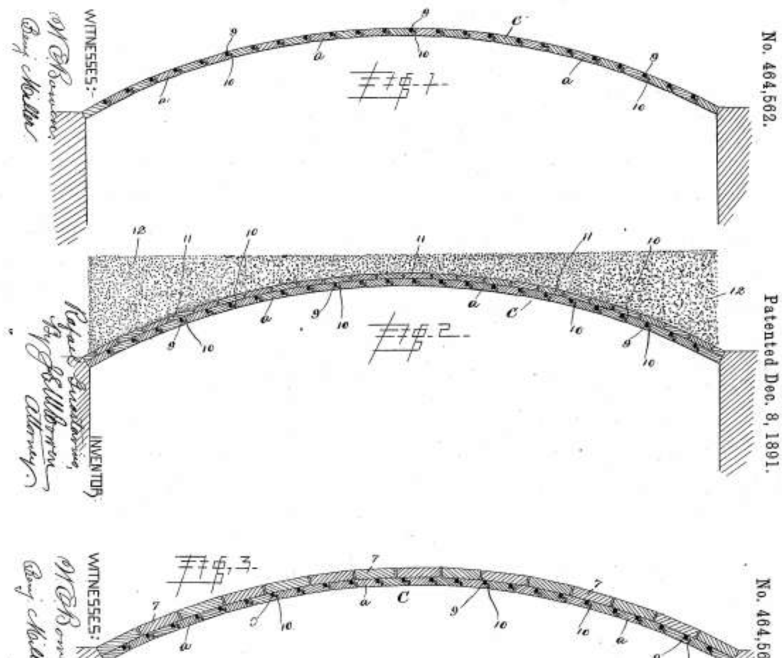


Figure 6 – Guastavino vault system (United States Patent office.)

The work of Rafael Guastavino in the United States was continued by his son Rafael Guastavino Expósito (Guastavino Jr.), he started working next to his father since the beginning of his career, in fact he supervised the construction of the Grace Universalist Church vault (Figure 7) of 21.3 m span and 150 mm of thickness with only 23 years old, for this dome Guastavino had to include steel bands in order to resist the hoop forces in the bottom, also in the intersection between the barrel vaults and the dome steel elements were provided.

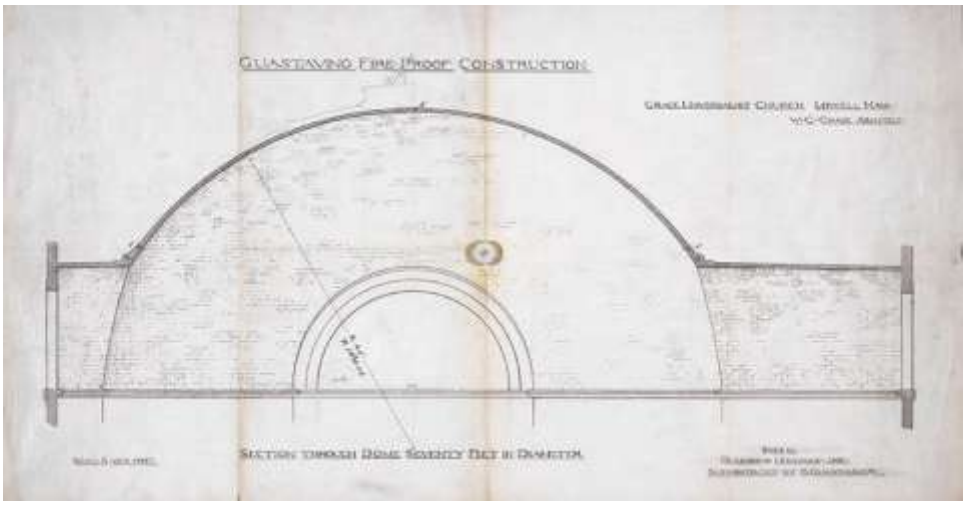


Figure 7 – Grace Universalist vault (Avery Library)

In 1892 Guastavino Jr. presented a new patent number 468,871 [8] in which included metallic anchors considered as a shear connectors between two layers of bricks or bricks with concrete, the objective of this anchors is allow the construction of composite vaults with concrete and cement (Figure 8).

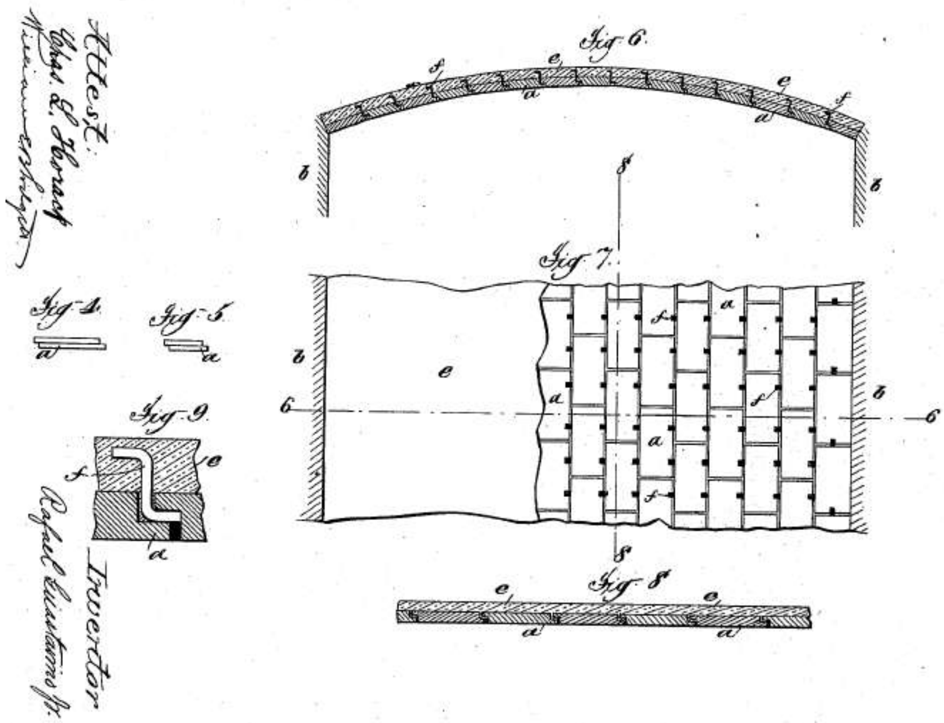


Figure 8 – Shear connectors in vaults, (United States Patent office.)

Guastavino Jr. continued to develop the idea of reinforced timbrel vaults, as an example new patents published in the XX century should be analyzed. In the Patent 947,177 of 1910 [9], he shows a reinforced dome (Figure 9) with metallic rods in parallel and meridian directions, according to Guastavino Jr. these rods are protected from fire and moist due are embedded inside the tiles.

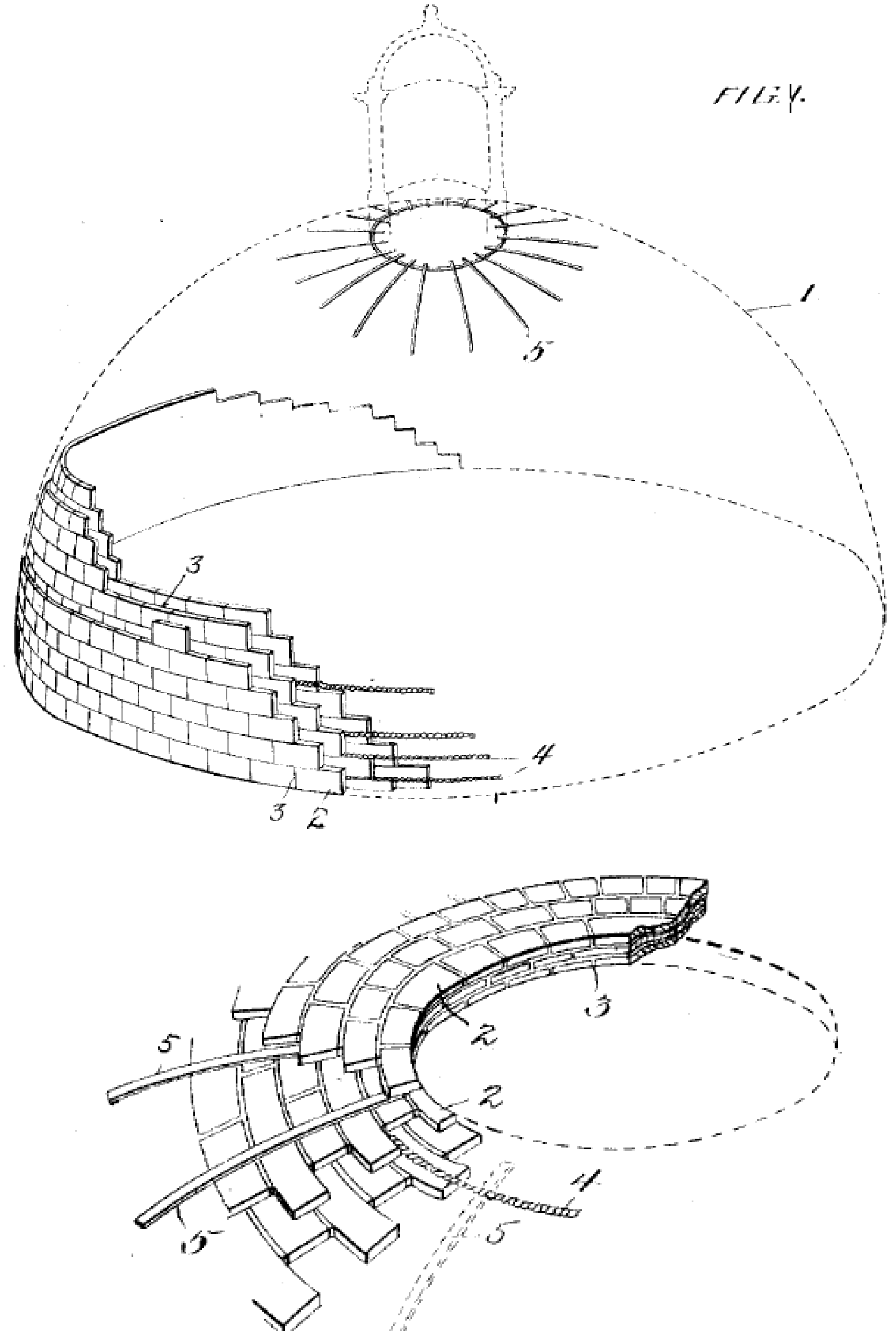


Figure 9 – Guastavino Jr. reinforced timbrel dome (United States Patent office)

In the same Patent a reinforced barrel vault is also shown (Figure 10) composite by tiles, cement mortar and metal rods, used for increase the strengthening of the vault, this rods are parallel to the barrel vault shape, in the perpendicular direction transverse rods were placed for prevent the separation of tile bricks. It's important to mention that the idea of reinforced masonry was published first at the end of XIX century in France by Paul Cottancin.

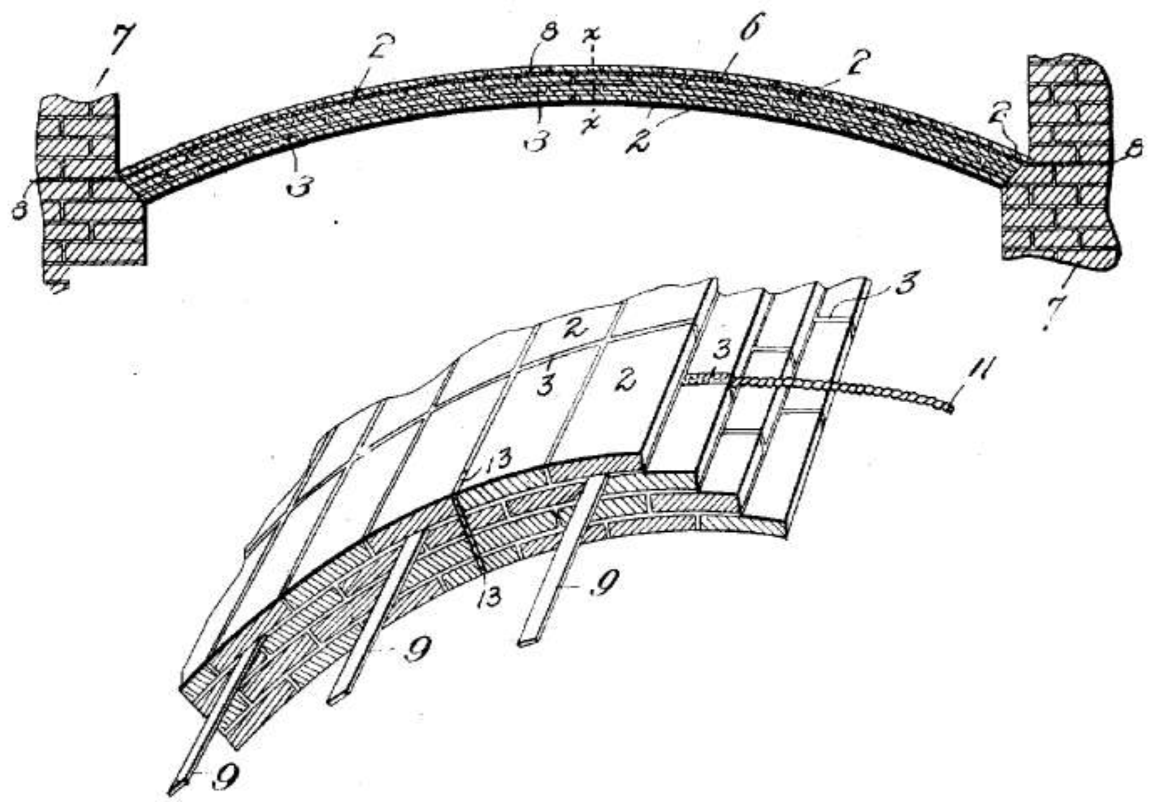


Figure 10 – Reinforced barrel vault (United States Patent office)

In 1913 the Patent 1,052,142 developed by Guastavino Jr. was published [10] in which a composite Timbrel - Concrete vault was presented, this vault was Composite by a lower section of tile bricks, as usual in timbrel vault and above them a layer of concrete is placed, different solutions with reinforcement were considered by Guastavino Jr. (Figure 11), it is interesting to observe that the reinforcement is present in the bricks and the concrete and also they are connected, maintaining the idea of connect this two different materials.

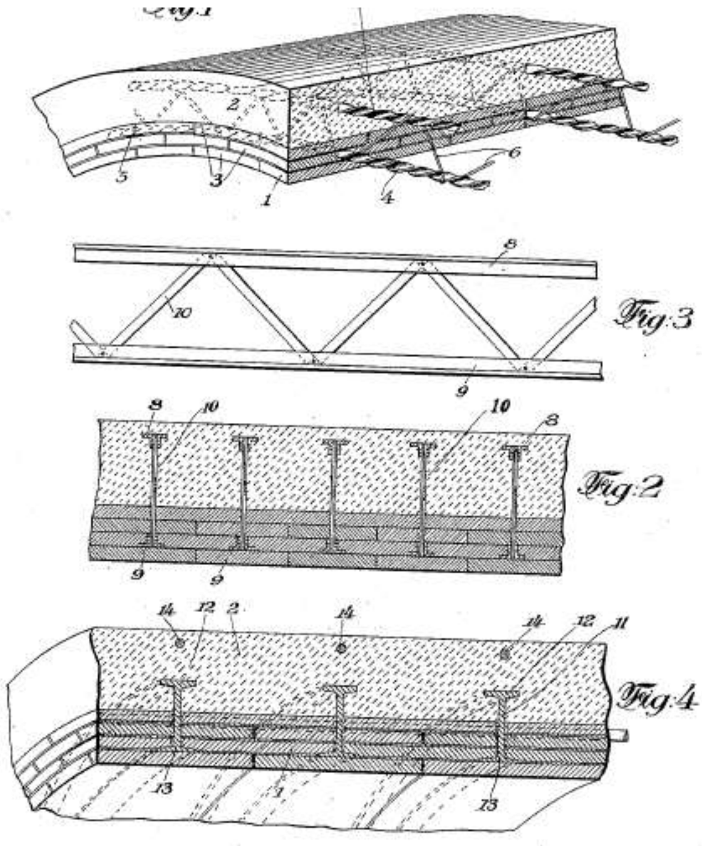


Figure 11 – Reinforced composite vault (Patent 1,052,142)

During Guastavino Sr. and Jr. developed the timbrel vault system in the United States, in Spain and Europe architects and engineers continued using the tile vault system. Santiago Rubió indicated in relation to the Catalan vaults that they are a kind of mosaic concrete that has enough elasticity to resist small deformations compatibles with the small embedment that occurs in the supports, he also developed a series of tests in vaults in order to justify their use in other countries where the system is not used, he mentioned a test developed in France in the hydroelectric power station of Saint Etienne where 200 x 150 x 20 mm bricks were used to built vaults with one and three layers, according his estimation the failure between 30 and 32 kg/cm^2 [11].

Antoni Gaudí also designed and built many timbrel vaults and at a certain point *inspired their mechanics* [11], as an example of the use of timbrel vaults can be observed in the church of the Colonia Guell, where he combined in one structure the use of different materials like concrete and tiles and different geometries in the shapes of the vaults like hyperbolic paraboloid.

Joan Bergós developed several test with different configurations, load patterns and steel reinforcement arrangements in timbrel vaults during the 40's of the XX century in Barcelona, and also calculated some vaults by using the funicular method, from his investigation about the structural behavior of this vaults he mentioned that hollow bricks and binders had a better behavior than the ones used by Gaudí [12]

Other important architect who worked with timbrel vaults was Luis Moya; he developed several constructions using this structural system like the chapel of the School Santa Maria del Pilar, where the structure is composed by a hyperbolic paraboloid vault built with 140 mm brick thickness and steel reinforcement in which only a falsework made of wooden planks separated at 600 mm was used. Another important contribution was the book named "Timbrel Vaults" in which explain several structure arrangements and constructive process. As a complement of Moya's book, Angel Truñó published in 1951 the book *"Construction of Tile vaults*" in which includes the use of the steel reinforcement in the vaults, especially in domes [13].

Eduardo Torroja was a Spanish engineer who also contributed with the development of the use and study of timbrel vaults. One of the most important constructions designed by Torroja was the church in Pont de Suert in 1952, three courses hollow bricks were considered and reinforcement steel was placed in the intrados [12].

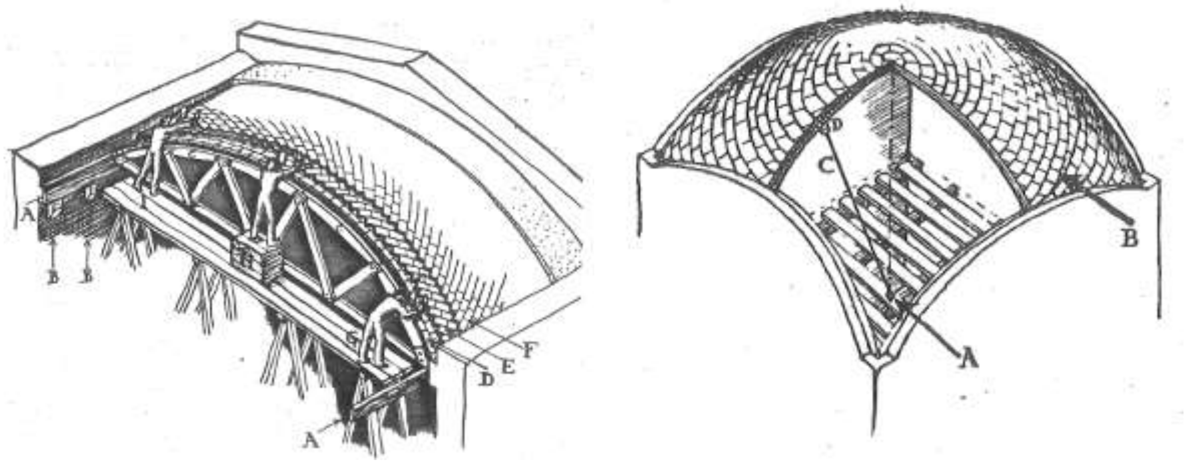


Figure 12 – Barrel vault construction by using a moving formwork (left) and a sail dome (right) (Moya 1947)

Le Corbusier also was another architect who was related with the timbrel vaults technique, for him the inspiration for use this structural system came from Gaudí, as an example in the Molol house of 1919 drawings Le Corbusier writes a note "Gaudí house" [14]. In spite of the inspiration for the use of timbrel vaults, he considered the tile layers only a formwork of the concrete, so he considered the concrete the only structural element of the vault. The most important reference of this system used by Le Corbusier is the Jaoul Houses.

Other examples of this structural system can be observed in South America, due the influence of Spanish architects and engineers like Eladio Dieste who proposed to use reinforced brick in several buildings, for example, the construction of the Jesus Obrero church in Atlántida Uruguay (Figure 13), built in 1960 who was also built in reinforced masonry. Parallel in Colombia Guillermo González Zuleta developed similar designs like the vaults of the Rayo Supermarket in 1955, with 22.5 m span m and 50 mm of thickness [15]. In Argentina Eduardo Sacriste was the exponent of the tile vault, influenced by Guastavino the Argentinean architect design several houses with this system also including the use of reinforcement and concrete to creating composite vaults [16].

Figure 13 – Jesus Obrero church in Uruguay, Eladio Dieste (Facultad de Arquitectura | Universidad de la República | Montevideo, Uruguay).

In XXI century new organizations have promoted the use of the timbrel vaults instead of the most used materials like reinforced concrete and steel. In the United States, the Masonry Research Group of the Massachusetts Institute of Technology, they developed l new tools for the design and analysis of masonry structures by using interactive equilibrium methods, thanks to this tools new shapes can be build by demonstrating their stability, this analysis of complex shapes were developed thanks to the computational software Rhino Vault (Figure 16) created by Philippe Block in 2009. One of the constructions built by this group was the 12 m span and 12 mm thick tile vault of the Conference Center Pines Calix in 2006 (Figure 14), who was the first timbrel vault in the United Kingdom [17].

Figure 14 – Conference Center Pines Calix (Carbon Free Group)

One of the examples of the use of form finding computational tools was the Brick - topia project [17] who was the winner of the contest for build a pavilion at the International Festival of Architecture Eme3, held in Barcelona in 2013, the structure had 4 meters height and 150 m^2 surface (Figure 15). The structure was supported in a concrete slab, as a formwork of the vault during the construction a system of scaffolding, cardboard sections and steel reinforcements was used; this is due to the complexity of the shape.

Figure 15 – Brick – Topia project in Barcelona (Lopez 2016)

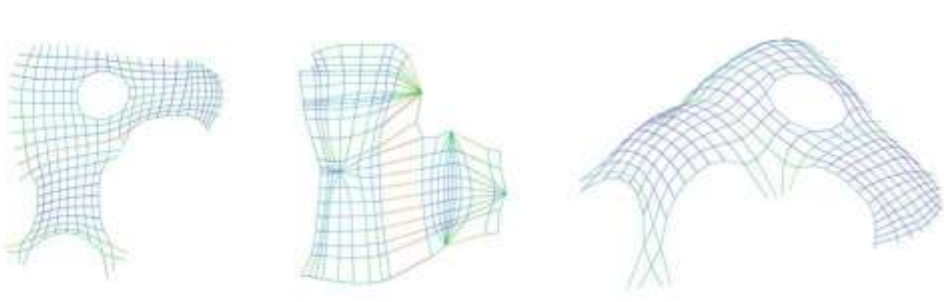


Figure 16 – Form finding in Rhino Vault (Lopez 2016)

4 STRUCTURAL ANALYSIS OF TIMBREL VAULS

The structural analysis of arches and vaults has been object of study since centuries, many authors have researched and written about this topic, is not the purpose of this document to generate a deep investigation of the contribution of each of them in the study of masonry arches. However, a brief review is important to see the different types of methods used in their construction.

As its knows, before the first rational approach developed first by Robert Hooke only geometrical rules were used for the calculation of arches and vaults based only in the experience of the builders without rational base. Between them the work of Alberti, Palladio and Fray Lorenzo was important.

The rational approaches began with Robert Hooke, who in 1675 indicates that one arch in the same way that a flexible thread hangs, but inverted, a rigid arch is stable. In 1697 David Gregory develops the mathematical calculations about the Hooke theory and also he states that the thrust force in a support will be equal as the force of the hanged thread but in opposite direction. La Hire tried to solve the problem of find the weight of each voussoirs that keeps the arc in balance by graphical methods, and he also found that the friction between the voussoirs need to be considered. Bélidor in 1729 indicates that the thrust force can be calculated as $2^{0.5}$ *W, where W is the weight of the arch. Couplet in 1730 establishes the following: the voussoirs have no tensile strength, the compression strength is infinite and the displacement between two voussoirs cannot occur. Another problem approached by Couplet was to find a relation between the radius of a semicircular arch and their thickness, and the result obtained is the thickness needs to be 10% of the semicircular radius. Poleni in 1748 used the hanged thread to resolve the Saint Peters dome in Vatican City. Coulomb in 1773 indicates, as a result of his investigation, that a minimum and maximum value of the thrust force can be obtained and the arch is stable if the thrust force value is between them [1].

The analysis of structures also follows other investigation areas. Navier in 1826 added the equations of elastic flexure to those of the statics to calculate the real state of a hyper static beam, for later, by means of the resistance of materials determine the values of tension in any point of a beam, he also indicated that the boundary conditions describe how the structure is connected to the medium in which it is located [18]. Castigliano in 1879 applied their theorems about the elastic energy in stone bridges taking into consideration the mechanical properties of the stone and mortar, he indicates that crack could be appear if the thrust line was outside the central third of the arch. Kooharian in 1959 demonstrates that the masonry arch can be analyzed with plastic theories. This theories changed the way of analyze the structures in the ends of the XIX century and the beginnings of the XX.

Jaques Heyman in 1966 published *The Stone Skeleton* in which considering the postulates of the rational approaches (infinite compression strength, nule tensile strength, no displacement between voussoirs) of the Coulomb theory is feasible. Jaques Heyman proposed the uniqueness theorem that consider that if there is a thrust line of one arch in equilibrium contained within the arch under external loads, that is close to develop enough hinges to transform the arch into a mechanism, the arch is about to collapse and this line is unique. Also defines the safe theorem what does it mean that if the thrust line is inside the arch, the arch is safe. The theorems of Jaques Heyman are a powerful tool for the arches and vaults analysis using nowadays for arch assessment.

According to timbrel vaults, the history of the structural analysis it's not much different from the arches, Fray Lorenzo focused in the thrust force in the walls and buttress dimensions, Espie in opposition mentioned that there is not thrust force because the vault is monolithic.

Centuries later, Rafael Guastavino Sr., published the first attempt to develop a theoretical background of the timbrel vaults, in addition, several test were developed to justify the security of the system (Figure 17). In his publication Guastavino classified the masonry structures in two groups: mechanical and cohesive in which the timbrel vaults belongs to the second group, according to him, the cohesive masonry has for basis the properties of cohesion and assimilation of several materials, thanks to this condition the vaults can be built without joints due the overlapping of bricks in the layers.

The only calculation that can be found in Guastavino's book is the thrust calculation of a flat arch or barrel vault, what is a equilibrium equation [6]

$$A\,(S_{br}) = W\,L/\,8h \tag{1}$$

Where A is the cross section area of the vault per unit of length, S_{br} is the breaking stress in compression (14 N/mm^2 according to Guastavino tests), W is the dead load (self weight + other permanent loads), L is the length of the span and h is the height of the intrados in the center. Guastavino define the working stress of the vault as 20% - 25% of the breaking stress; it is important to indicate that the surface A is in the center of the vault (crown) and it is bigger near the supports and can be calculated with the formula of the French Dejardin. He Also hired a professor of applied mechanics from MIT named Gaetano Lanza to develop an Elastic analysis of the vaults by obtaining the stresses in the vault considering the axial force and bending moment. Despite of the intentions of Guastavino Sr. to give a theoretical background to their system, he based their calculations in mechanical arches without considering the cohesive behavior, nevertheless there is no doubt about his capacity as a vault builder.

Figure 17 – Load test in Guastavino vault in 1901 (Avery Library, Columbia University)

Guastavino Jr. also tries to develop a structural analysis for timbrel vault, he used Graphic methods for the design of the vaults and also the work of Dunn published in 1904 for domes.

The apparition and the increase in capacity of computers during the XX century allowed the use of new techniques for complex equations related with the structural analysis like the Finite Element Analysis, in which the equations that govern elasticity problems are considered, the ability to perform three dimensional models is an improvement over the two dimensions limit analysis. In the case of masonry structures the elastic analysis has not significance and do not simulate the real behavior of the structure, specifically in the case of timbrel vaults the support conditions, previous crack of the masonry and load pattern are a important variables to take into consideration.

The apparition of non linear analysis in the 1980's allows developing more precise macro-modeling approaches which can include geometrical and material nonlinearities, tensile and compressive strength of the materials and allows studying even the collapse of the structures. In the XXI new analyses were developed in timbrel vaults by using Finite Element Models, a description of them is presented in the following paragraphs.

In 2003 Saliklis [19] published “*Finite Element of Guastavino tiled arches*”, in their work tiles fragments obtained from Guastavino vaults were tested by using non-destructive and mechanical tests in order to obtain the mechanical properties. The results of longitudinal a transversal dynamic tests and monotonic compression and flexure testing were: 21520 N/mm^2 Young’s modulus in x-direction and 12000 N/mm^2 Young’s modulus in y-direction, the average value of Young’s modulus was 16548 N/mm^2, the compressive strength measured was 34 N/mm^2 in x-direction and 23 N/mm^2 in y-direction and the flexure strength was 11 N/mm^2 in x-direction and 5 N/mm^2 in y-direction. The Young’s modulus of the mortar considerer was 689 N/mm^2 (Figure 18).

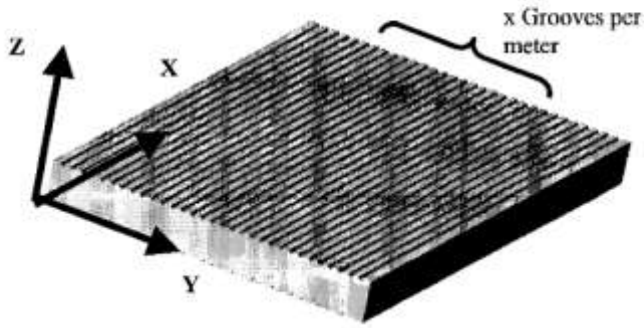


Figure 18 – Coordinate system considered in the analysis (Saliklis 2003)

From these properties, three arches with different spans were modeled in Finite Element Method by using the software ANSYS. On it; mortar and tile were modeled by using eight-node isoparametric laminated shell elements. As a result of the Finite Element Model, the authors obtained a maximum compressive stress of 5.5 N/mm^2, which is lower than the values obtained by the tests and Guastavino; also they demonstrated that influence of the Young’s Modulus of the Mortar was not relevant.

In 2007, Sezer Atamturktur and Thomas E. Boothby published “*The Development of Finite-Element Models and the Horizontal Thrust of Guastavino Domes*” [20]. In this study two domes built with Guastavino system and designed by Henry Hornbostel were considered, this corresponds to the City-County Building entrance-vestibule domes in Pittsburgh, and the New York State Education Building Reading Room domes in Albany. The mechanical properties used were those obtained by Salikis [19] for Young’s modulus and also samples were collected and tested in laboratory, experimental modal analysis (EMA) was also used to obtained the natural frequencies and modes of vibration of the existing domes. With all this information, a Finite Element Model was developed in order to calibrate the mechanical properties with those obtained by the experimental analysis. From the tests, a Young modulus value of 13200 N/mm^2 in the longitudinal direction and 15400 N/mm^2 in the transversal direction were obtained for the tiles and for the mortar a value of 2970 N/mm^2 was obtained, from this values an effective Young’s modulus is obtained by a homogenization procedure, the resulting value was 7600 N/mm^2 and the density was 1800 kg/m^3. The thicknesses of the domes were obtained by using the impact-echo technique and the value obtained was approximately 135 mm.

The two domes were modeled in the software ANSYS by using shell elements, these models were used for the calibration of the support conditions for later develop a new non-linear model in SAP 9.0 to know the behavior of the domes, especially regarding with the thrust forces. As a result, the Finite Element Model revealed the significance of the lateral thrust in domes, according to Guastavino the horizontal thrust of the domes is 13% of the weight of the dome but the results obtained shows thrust forces over 55%.

In the context of the Brick – topia project, the complex shape designed with Rhino vault was modeled by performing a lineal analysis with Finite Element Model. The values of the mechanical properties considered were 3200 N/mm^2 Young's modulus, 0.15 Poisson ratio, 1219.4 kg/m^3 density, 0.24 N/mm^2 tensile strength and 5.90 N/mm^2 for the compressive strength. The results of this analysis can be observed in Figure 19, in which the compressive stresses for the most unfavorable load combination are shown, from the point of view of the compressive capacity the bricks the maximum stress obtained is much lower than the maximum value, so the thickness of the vault is verified.

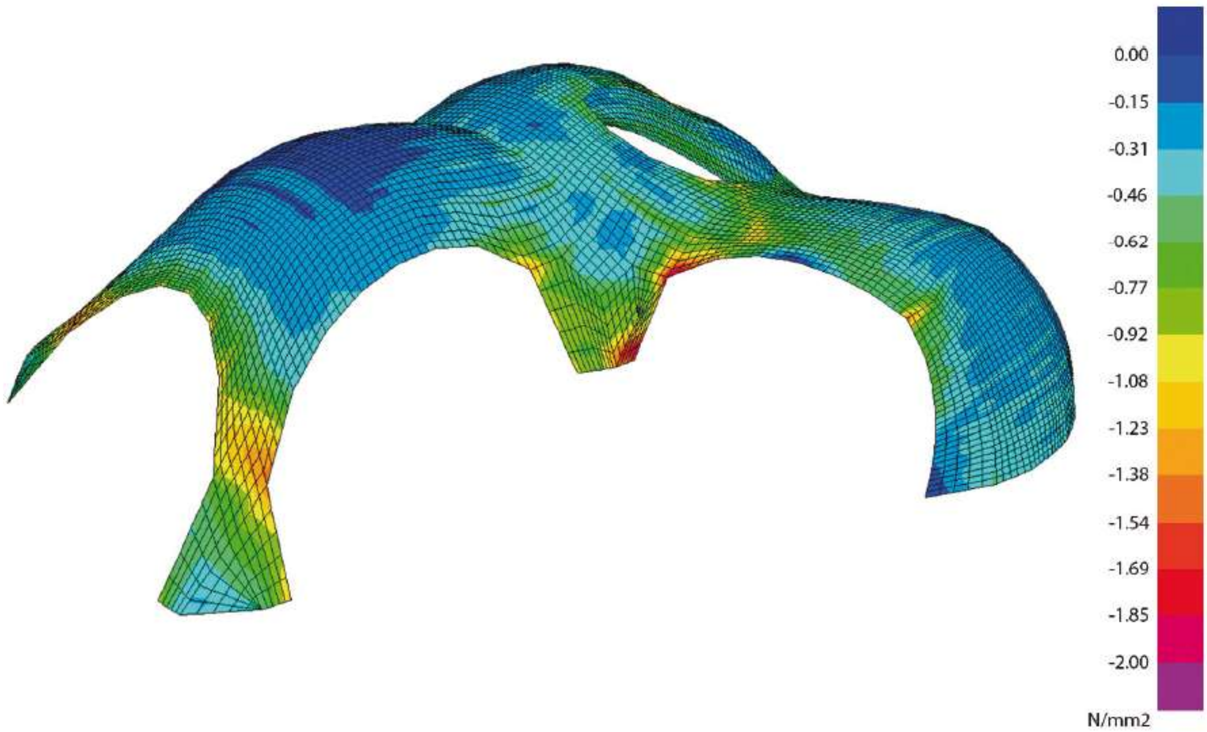


Figure 19 – Linear analysis Finite Element Model for the Brick – topia structure (Lopez 2016)

In 2014, João C. M. Rei and António S. Gago od the University of Lisbon also worked in the study of timbrel vaults, their work was published in 2017 titled *Timbrel vault - a traditional constructive technique [*21]. In this, a full scale timbrel vault was built and tested and Discrete Element Models were developed by using the software 3DEC (Figure 20).

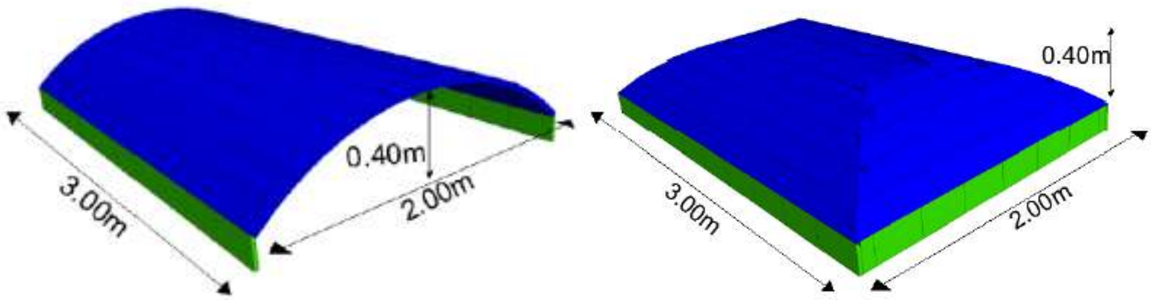


Figure 20 – Discrete Elements Model of timbrel vaults: Barrel (left) and cloister (right) (Rei 2017)

Figure 21 – Prototype of cloister vault built in the University of Lisbon laboratory (Rei 2017)

The prototype was loaded by placing precast paving slabs above it with two configurations, uniform and asymmetric in one side of the vault, before that sand bags were placed to ensure the correct load transfer into the vault. For the uniform load, 5 kN/m^2 were applied and the vertical displacement in the key zone was 7 mm. For the asymmetric load, the precast paving were placed until the collapse that occurs at 7.7 kN/m (Figure 22), this value is higher than the obtained from the Model. The authors explain this difference due the influence of the fill in the prototype, which is not considered in the model.

Figure 22 – Prototype of cloister after the collapse (Rei 2017)

The distribution of the thrust forces is evaluated for the long and short edge of the cloister vault, this was also analyzed for three different load positions (y = 0.000 m, y = 0.358 m and y = 0.781 m where y is measured from the longitudinal axis) also the Self weight (PP) is evaluated (Figure 23).

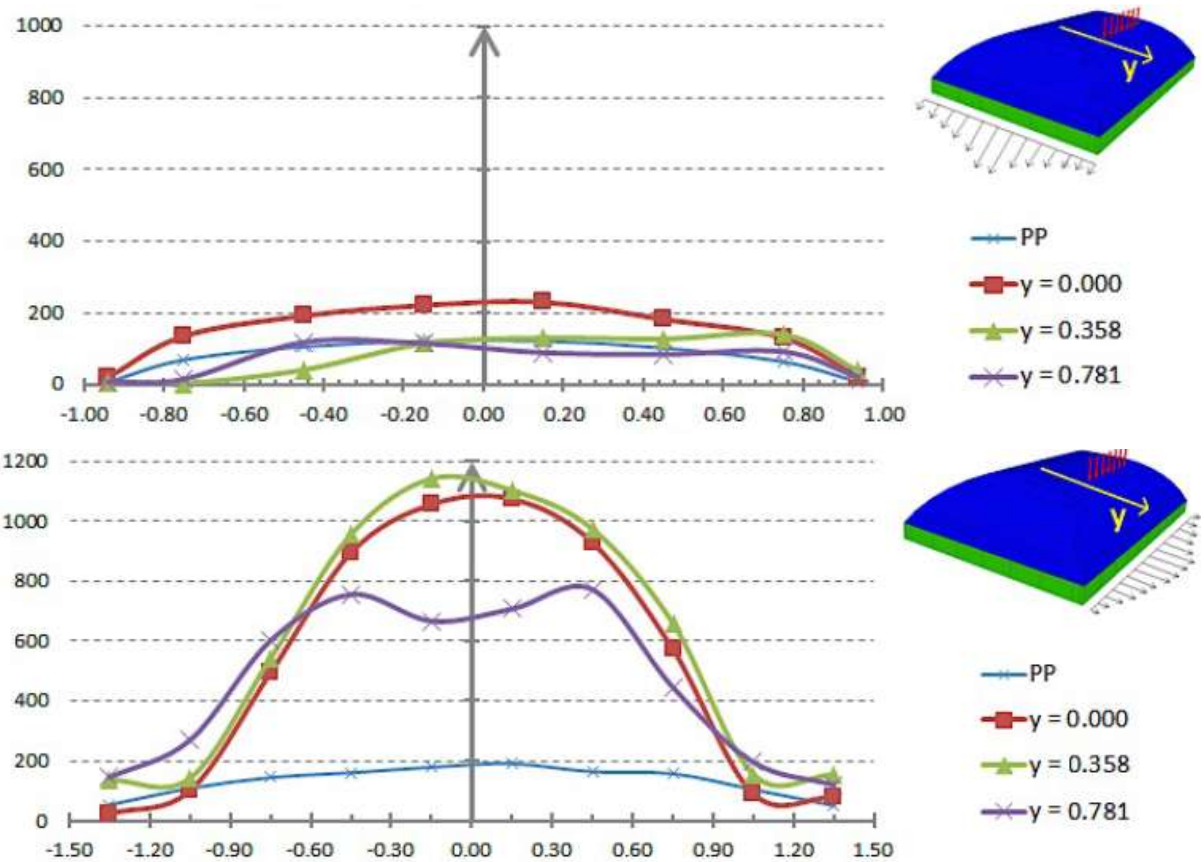


Figure 23 – Thrust force distribution along the edges of the cloister vault model (Rei 2017)

For the Barrel vault the collapse load was analyzed for different load positions, the model 1 and 2 corresponds to heights to the key point of 0.4 m and 0.6 m.

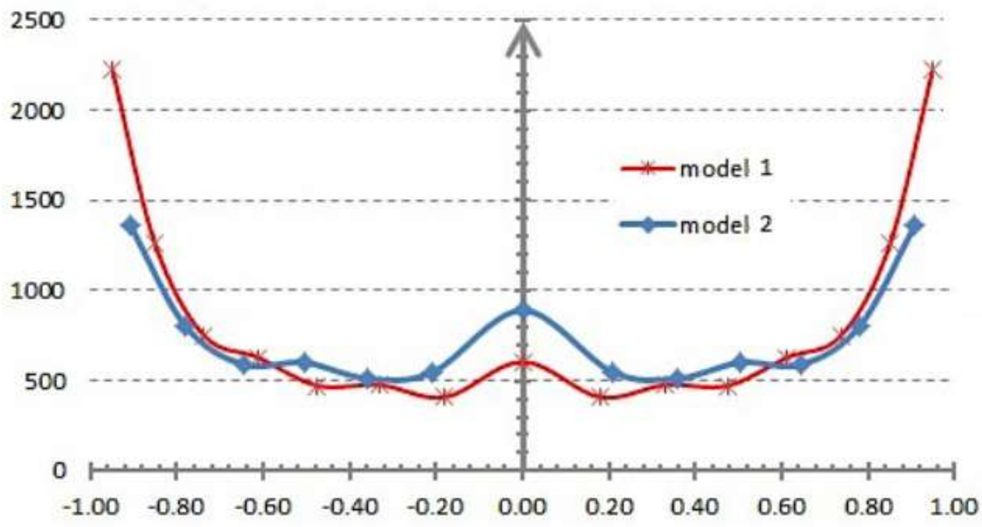


Figure 24 – Thrust force distribution along the edges of the cloister vault model (Rei 2017)

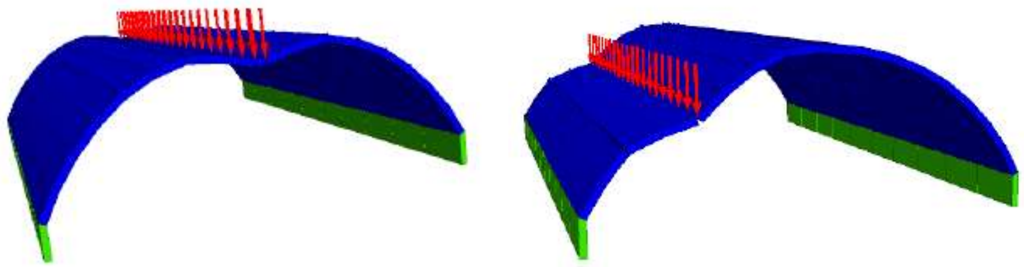


Figure 25 – Collapse mechanism for the barrel vault model (Rei 2017)

Another structural analysis of timbrel vaults was published in 2015 by David Lopez and Marta Domènech named "*Tile vaults. Structural analysis and experimentation"* [22]. In this work, a non-linear analysis with macro model approach with Finite Element Methods were performed in 2 dimensions, the self – weight and other external load were applied in a length of 40 cm at a quarter of the span. The mechanical properties considered were the same considered in the Brick - topia model but considering in this case a non-linear properties were needed like the fracture energy, the values used were 0.14 N/mm for tension and 9.44 N/mm for compression. The stresses at the collapse mechanism of the vault is shown in Figure 26

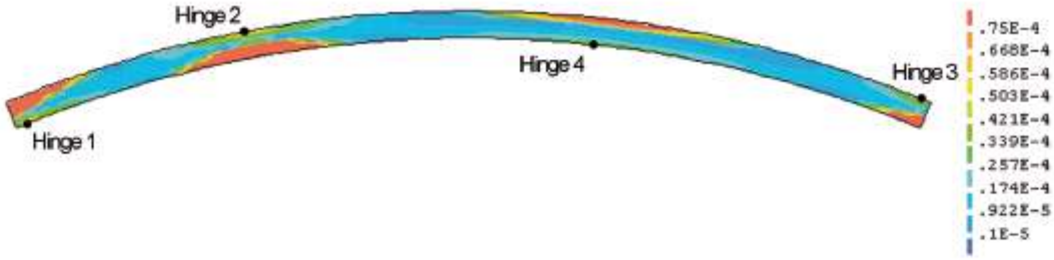


Figure 26 – Collapse mechanism for the vault model (Lopez 2015)

A micro-model approach for the non-linear analysis was analyzed; the units were modeled as elastic continuum elements, the joints were inelastic interface element and the composite yield surface was defined by a function. In this case more information about the mechanical properties of each material was recollected. The stresses at the collapse mechanism of the vault is shown in Figure 27

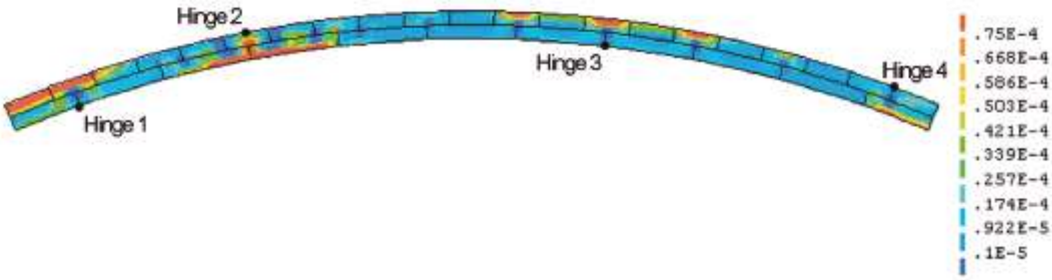


Figure 27 – Collapse mechanism for the vault model (Lopez 2015)

Both models were compared and the results were very similar (Figure 28), but both differ from the experimental results, in any case the experiments differ from each other and the model results are in the middle between both results. One of the conclusion of the this investigation is that the use of macro models gives good results and requires less information and resources.

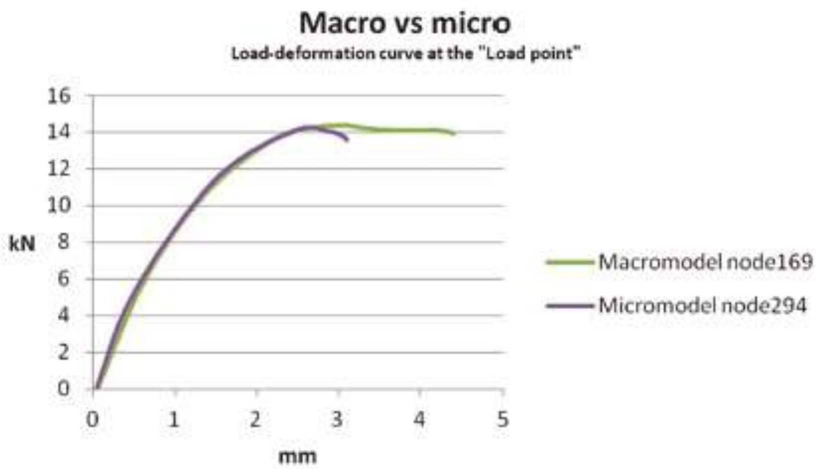


Figure 28 – Comparison between macro and micro modeling (Lopez 2015)

Besides, three vaults were studied with different configurations: three layers of bricks and two layers of bricks with stiffeners (Figure 29). The dimensions considered were 1 meter width, 3 meters span and 0.3 meters heights. For this case 2D, and 3D models were developed. The vault with two layers was named reference vault.

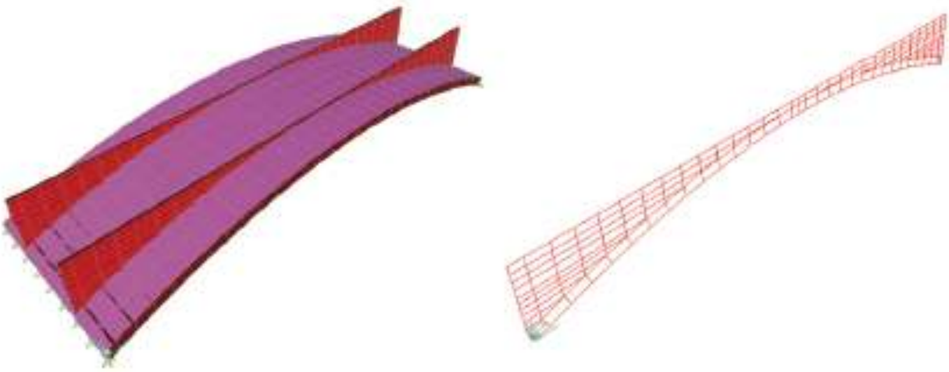


Figure 29 – 2D and 3D model of vault with stiffeners (Lopez 2015)

The comparison of the results is shown in Figure 30, it can be observed that both configurations but the third bricks of layers increase significantly the capacity of the vault, it is important to mention, as can be seen in Figure 29, that the stiffeners are not connected to any support, which would increase the capacity if they were.

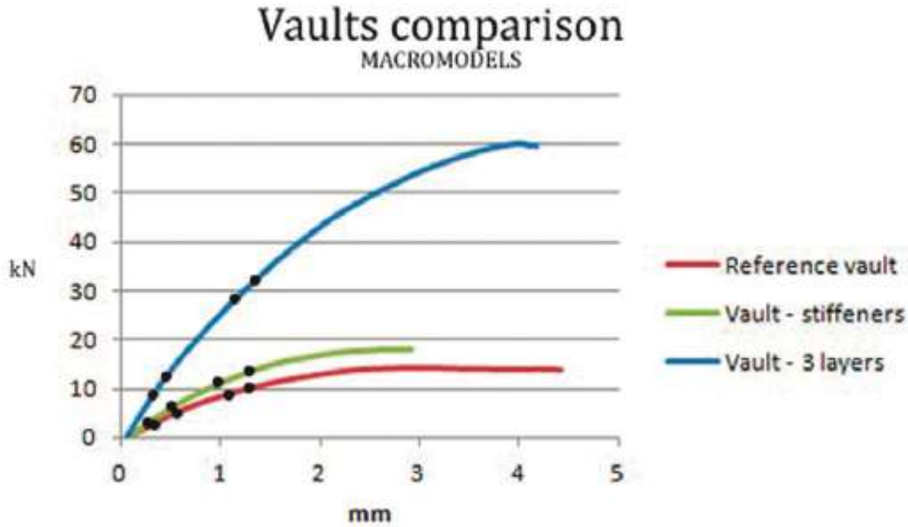


Figure 30 – Comparison of the vault macromodels (Lopez 2015)

In 2018, John Dhaenens from the Ghent University published *Exploration of the abilities of free – handed masonry vaults* [23]. The objective of this work was to analyze the stability of timbrel vaults and domes without using formwork during their construction and in its final condition, to this purpose; a non-linear Finite Element Analysis in ABAQUS was used. Different shapes configurations, thickness of elements and density materials were evaluated, as an example can be observed in Figure 31. From their results, he conclude that that the stability cannot simply be related to specific geometrical requirements, also the previous indicated parameters need to be considered in the analysis.

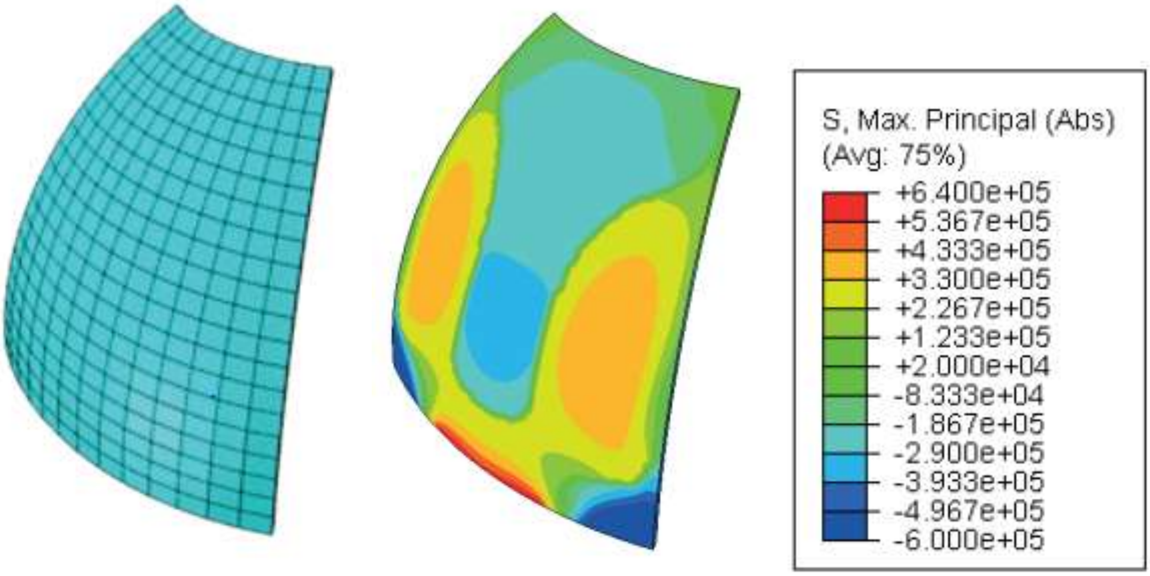


Figure 31 – Maximum stresses on Finite element surface. (Dhaenens 2018)

Finally, David Lopez and Pere Roca [24] develop the Extended Limit Analysis for Reinforcement Masonry (ELARM) published in the paper *Tile vaults as integrated formwork for reinforced concrete: Construction, experimental testing and a method for the design and analysis of two dimensional structures*. This method of analysis can be used for 2D Composite Timbrel - Concrete vaults. In this method, the moments are calculated from the axial load produced by the external loads and from the self weight, from the maximum moment and axial load, the eccentricity is calculated. The structure is divided in a series of virtual voussoirs and also the thrust line can be analytically defined.

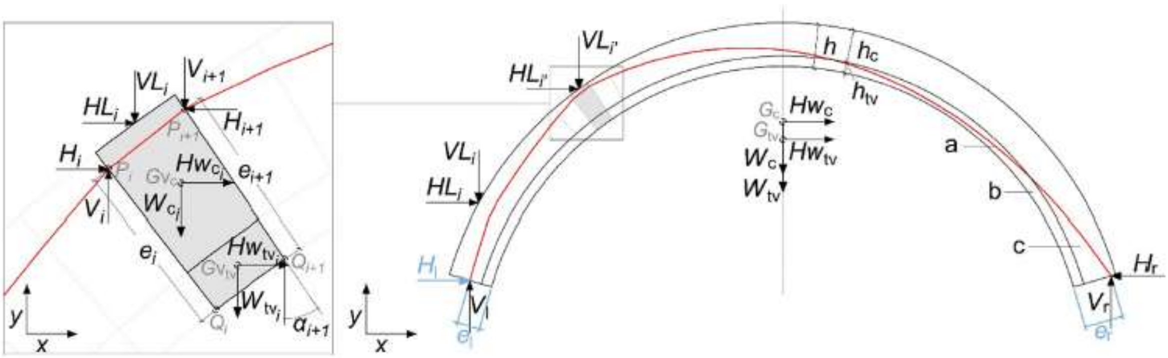


Figure 32 – Forces in a virtual voussoir and in the entire arch for ELARM analysis (Lopez 2019).

5 LABORATORY TEST

5.1 Overview

With the purpose of analyze the behavior of this construction system, an experimental research was carried out in the Laboratory for Technological Innovation of structures and Materials (LITEM) of the Polytechnic University of Catalonia (UPC) by David Lopez [24]. Their research include the test of full-scale prototypes with different configurations until failure, the prototypes tested were tile barrel vault (Figure 33), Composite (Tile - Concrete) barrel vault (Figure 34), and Composite sail dome (Figure 35). Two prototypes were built for each configuration.

Figure 33 – Tile barrel vault prototype (David Lopez)

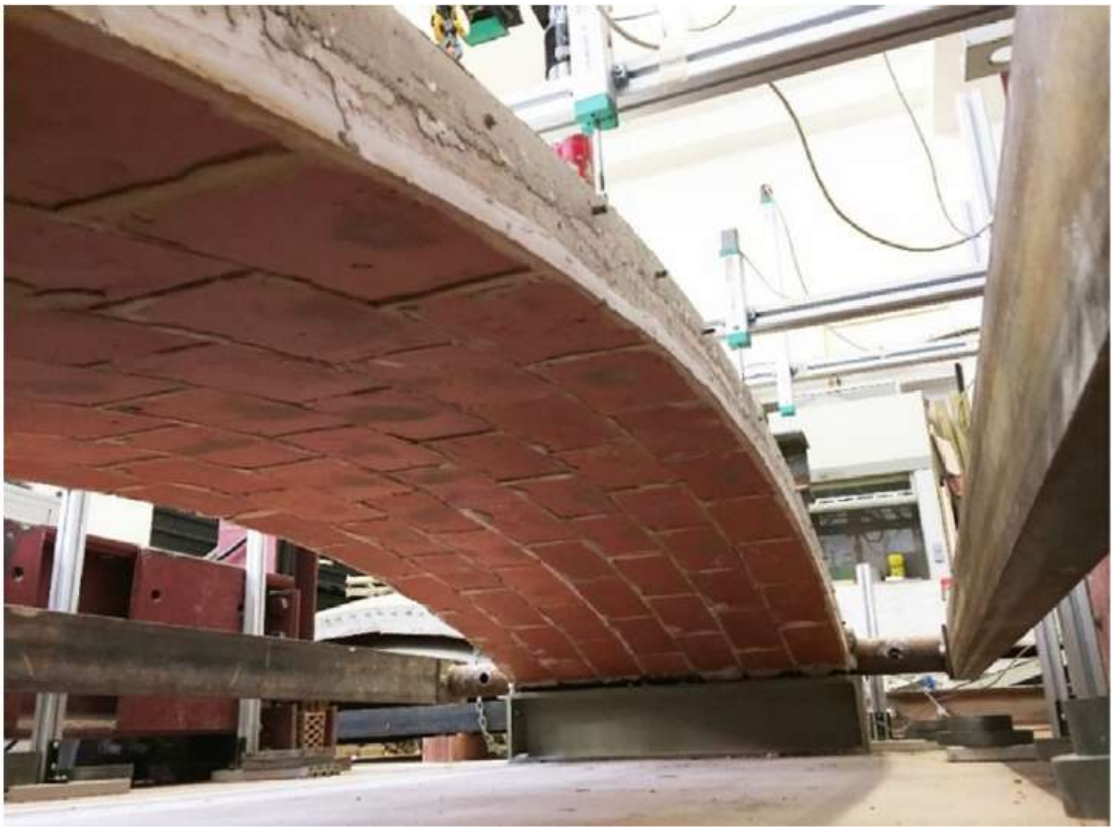


Figure 34 – Composite barrel vault prototype (Lopez 2019)

Figure 35 – Composite sail dome prototype (David Lopez)

5.2 Geometry

In relation with the Geometry of the prototypes, this is defined from the intrados of the vaults that means the span inner length and the inner height, the thickness depends on the layers of masonry and concrete. For the barrel vaults (tile, composite) the width considered was 1 meter.

For the timbrel barrel vault, the span was 2.80 meters, and the height was 0.25 meters. The thickness of the vault was 36 mm and considered 2 courses of bricks and 1 layer of mortar which are described in point 5.3. The load application point is placed at 25% length of the full span that is 0.7 m from the support and had a width of 115 mm; this load is applied along all the vault width. A scheme of the prototype dimensions can be seen in Figure 36.

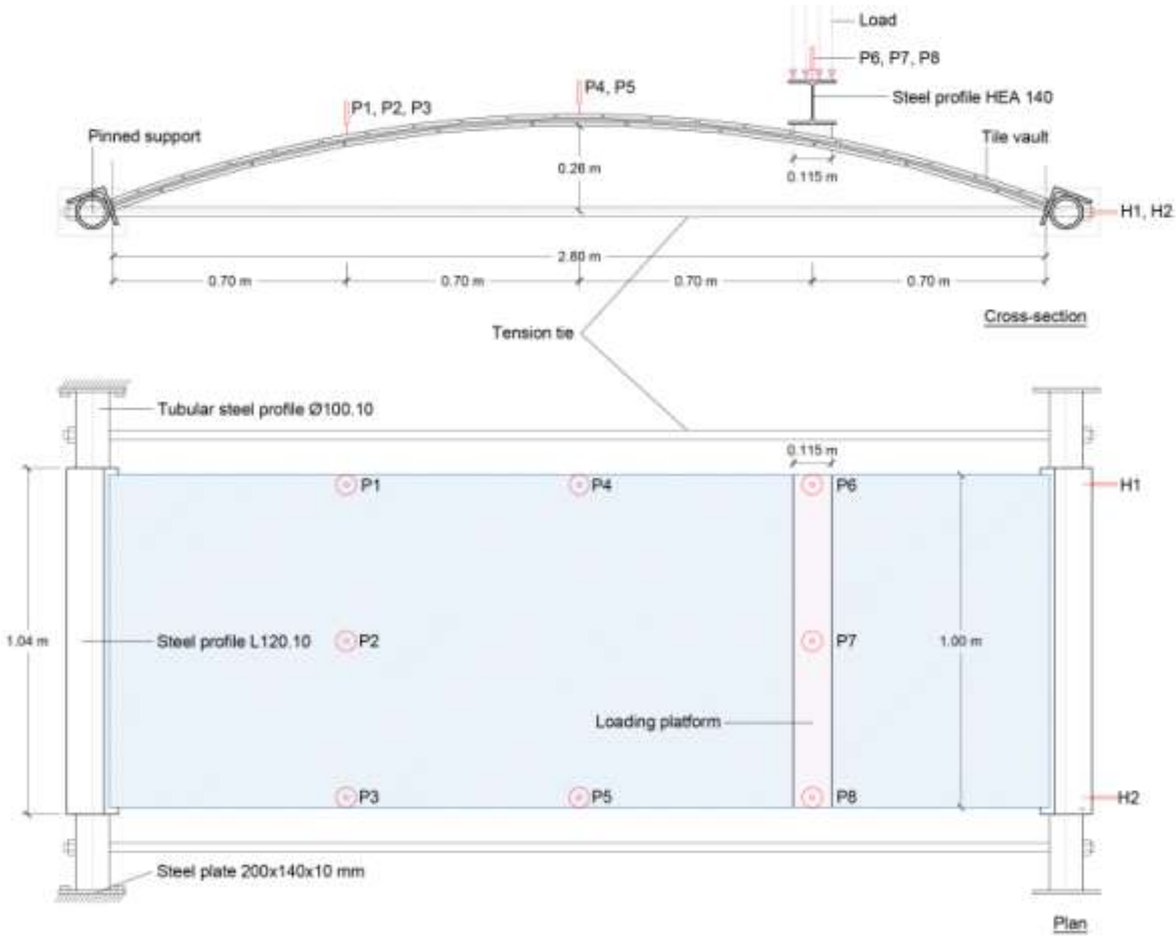


Figure 36 – Tile barrel vault prototype geometry (Lopez David)

For the Composite barrel vault, the two prototypes had the following dimensions; the span length was 2.78 m and 0.25 m height, the thickness of the vault is 36 mm for tile masonry and 50 mm. For reinforcement, 6 mm diameter steel bars were spaced at 70 mm in both directions so 14 bars were placed in the short side and 41 bars in the large side. The load application point is placed at 25% length of the full span that is 0.695 m from the support and had a width of 115 mm; this load is applied along all the vault width. A scheme of the prototype dimensions can be seen in the following Figure 37.

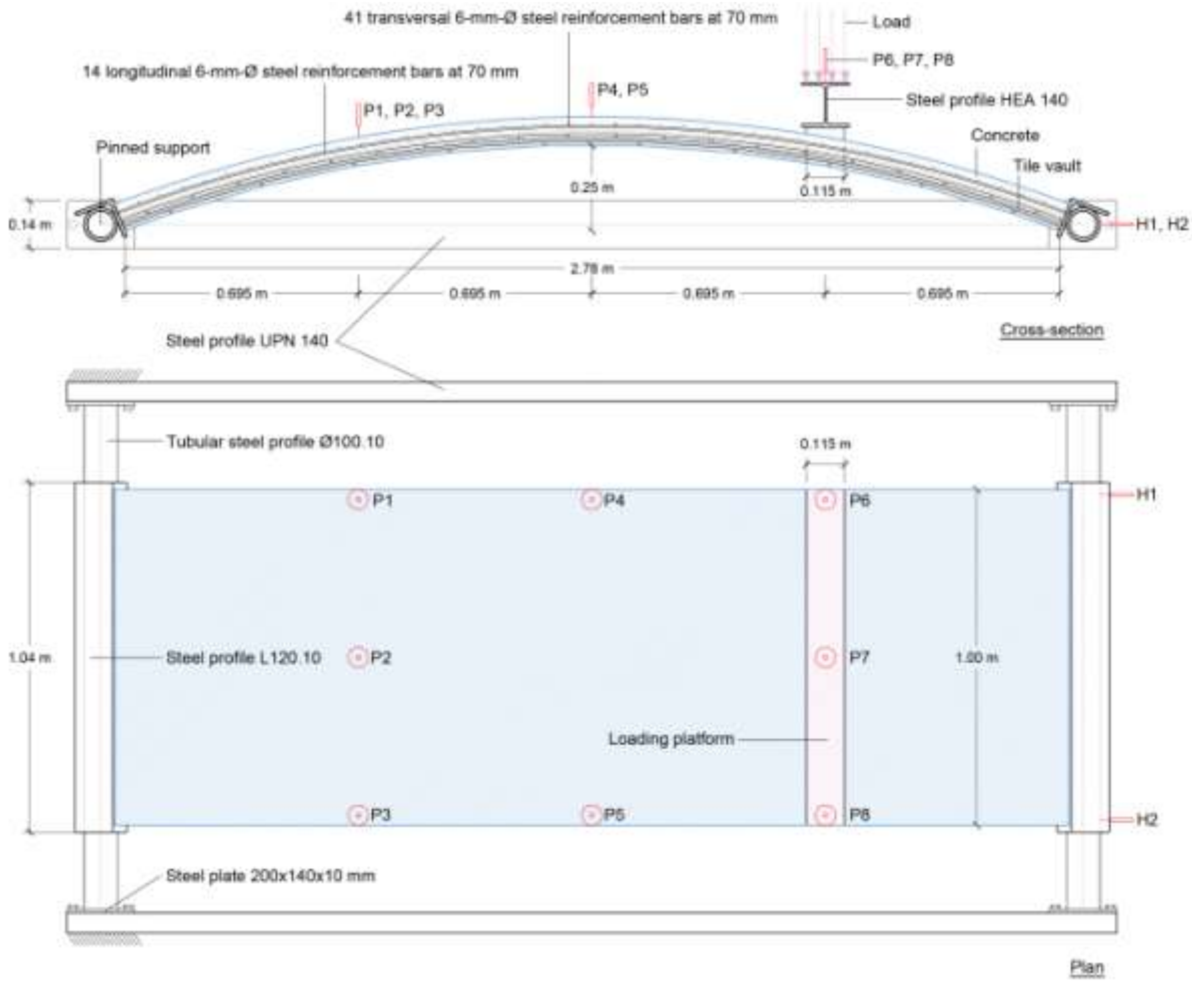


Figure 37 – Composite barrel vault prototype geometry (Lopez 2019)

For the sail dome, a square shape was considered with 1.76 m length for each four sides, the height until the upper part of the intrados was 0.39 m; the thickness of the vault is 95 mm. For reinforcement, 6 mm diameter steel bars were spaced at 70 mm in both directions so 27 bars were placed in each directions. The load application point is placed at 25% length of the full span in each side, which is 0.44 m from the support and had a square shape of 0.16 m for each side. A scheme of the prototype dimensions can be seen in the Figures 38, 39 and 40.

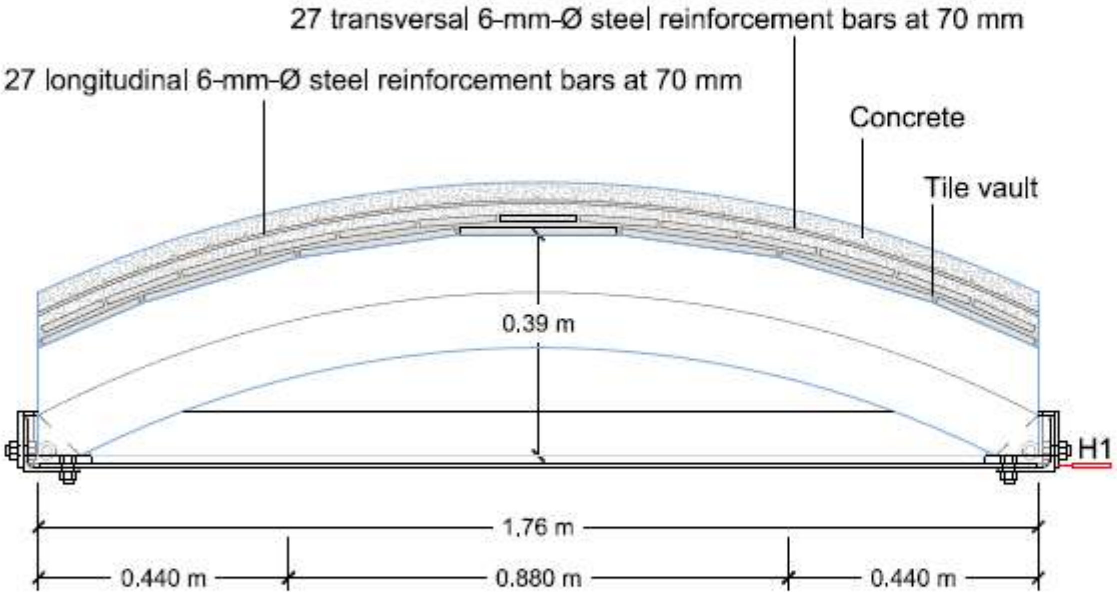


Figure 38 – Composite sail vault prototype cross section geometry (David Lopez)

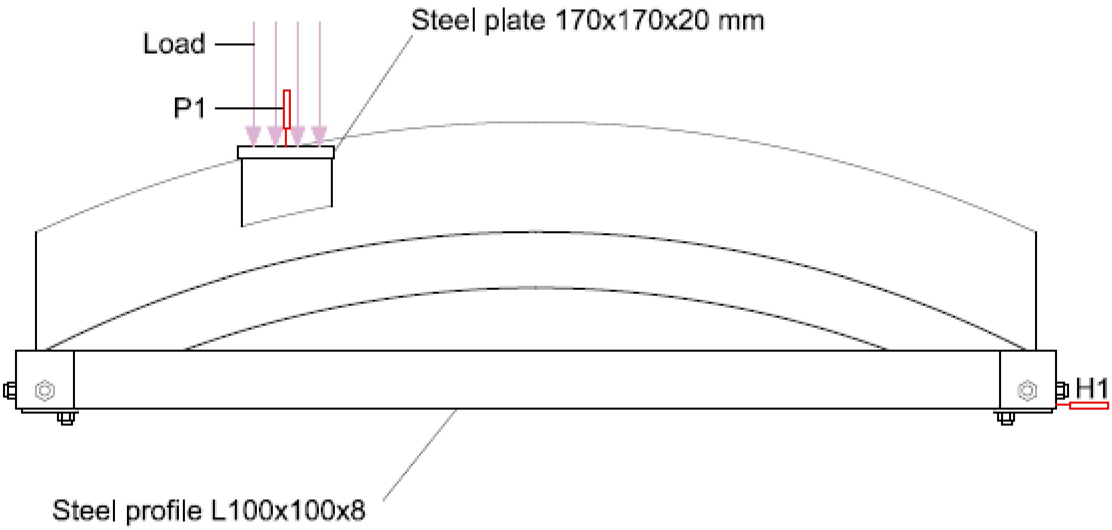


Figure 39 – Composite sail dome prototype elevation geometry (David Lopez)

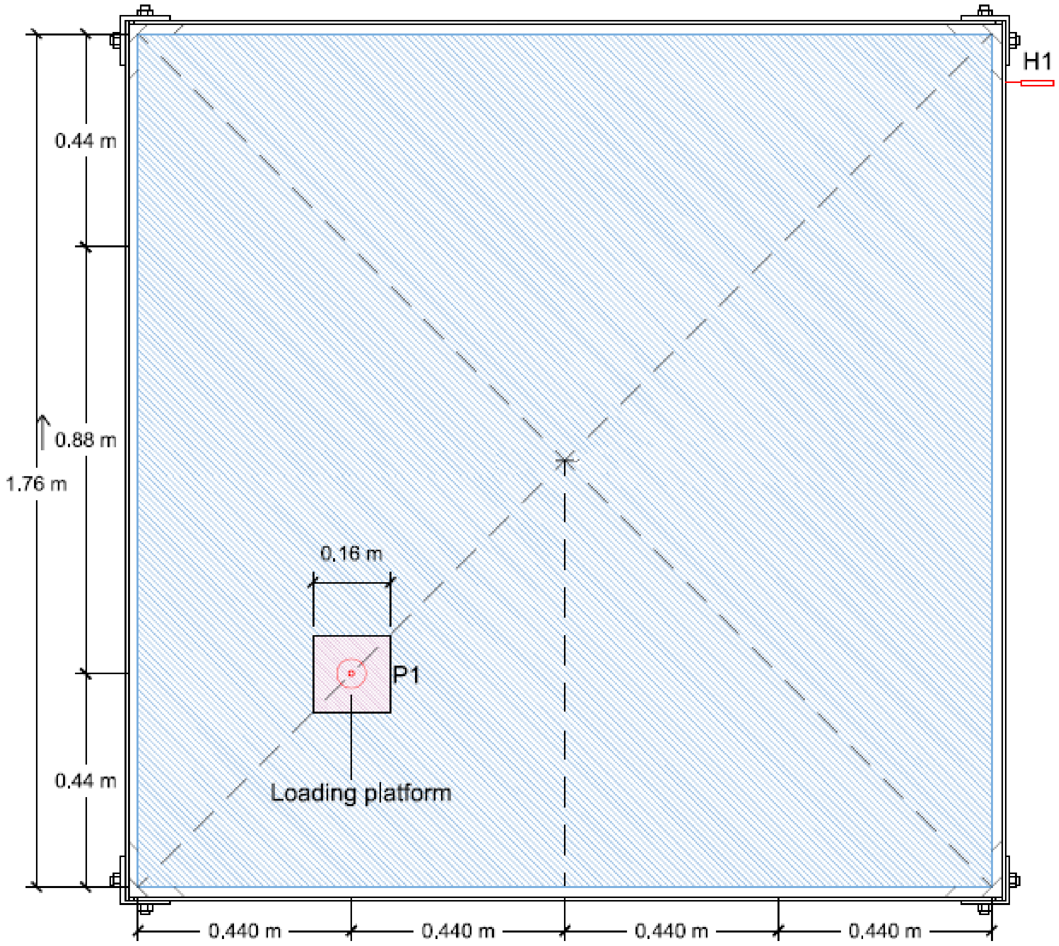


Figure 40 – Composite sail dome prototype plan view geometry (David Lopez)

5.3 Materials and Mechanical properties

The materials used for the construction of the prototypes were tiles, mortars, concrete and steel reinforcement.

For the tiles the units had sizes of 277x134x13 mm, along the thickness two configurations can be observed with 6.5 mm with grooves and the other 6.5 mm filled along the width. The mortar used was fast setting cement for the first course of tiles, for the second course of tiles and for the joint between the courses Portland cement mortar was used, the thickness of the joint was 10 mm. The first course is considered as the one can be seen from the intrados and the second course is in contact with the reinforced concrete, in both courses the grooves are oriented towards the joint.

In order to obtain the mechanical properties of the components of the vault, several test were carried out, the values obtained came from the tests developed for the Composite barrel vault and described by David Lopez [24]. The test procedure is shown in Figure 41.

Figure 41 – Test in both direction of bricks (left and center) and mortar (right) (David Lopez)

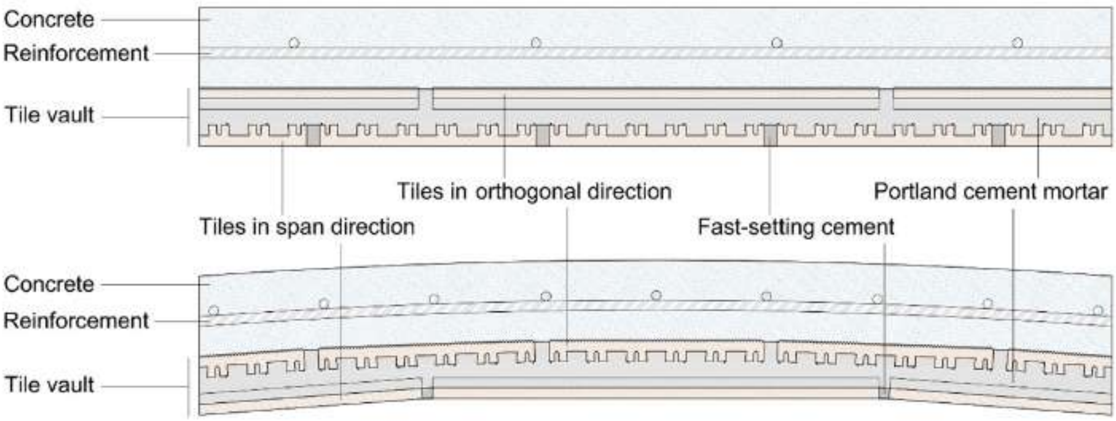


Figure 42 – Cross section of the vault in both direction and arrangement of tiles (Lopez 2019)

The tiles were tested in compression in the longitudinal and transversal directions; this is because the arrangement of the tiles is perpendicular to the span length in the first course and parallel to the length in the second course. For each direction, four compression tests were carried out and the results are shown in Table 1.

Table 1 - Mean values of compression strength tiles (Lopez 2019)

Longitudinal (first course) N/mm^2	Perpendicular (second course) N/mm^2
111	87

Regarding with the unit weight of the courses, nine masonry samples were measured and weighed and the result was 2000 kg/m^3. In addition, fast setting cement used in the first course and Portland cement mortar used in the second course and the joint between courses were tested inn compression, the result are shown in Table 2.

Table 2 **–** Mean values of compression strength mortars (Lopez 2019)

Fast setting cement (first course) N/mm^2	Portland cement mortar (second course and joint) N/mm^2
4.47	6.98

For the concrete, ten 100 mm cubic samples were produced, and tested in compression; the mean value of compressive strength obtained was 27.75 N/mm^2. As the masonry, the samples were measured and weighed in order to obtain the density; the mean value obtained was 2460 kg/m^3.

Finally, ten 6-mm-diameter reinforcement steel bars were tested in tension with the purpose to obtain the yield stress and Young's modulus. The mean values obtained are shown in Table 3.

Table 3 – reinforcement steel (Lopez 2019)

Tensile yield strength N/mm^2	Young's modulus N/mm^2
581	207000

5.4 Supports and loading

For the barrel vaults, pined supports were defined by using round and angle shaped steel elements that allow the rotation at the both edges, during the construction the edges were blocked against rotation [24] the support system for the barrel vaults and sail dome is shown in Figure 43 and 44.

Figure 43 – Barrel vaults support system (Lopez 2019)

For the sail vault, for supports were placed in the corners of the vault and acts like a simple supports and also only resist compression forces.

Figure 44 – Sail dome support system (David Lopez)

In all the vaults, a vertical load was applied by a hydraulic jack in the places described in Point 5.2. The test was carried out under displacement control at a constant speed of 0.4 mm/min.

For the data acquisition, besides the applied load a series of displacement transducers were placed in all the vaults, the position of them can be observed in Figures 36, 37, 38 and 39 but in general for barrel vaults were placed at 25%, 50%, 75% of the span length and one of the supports, for the sail vaults are only located in the load application point and one of the supports.

5.5 Results

For each prototype tested, load – displacement curves were obtained, for both vertical and horizontal loading. For the tile barrel vaults the results are shown in Figure 45 and 46, for the vertical load in vault 1 a linear behaviour can be observed until the peak point is reached, the load value obtained is 4.319 kN, the displacement at this point was 3.135 mm, this point corresponds with the third hinge apparition (the two first hinges were placed in the supports), the load decreases until 3.492 kN showing a brittle behaviour of the tiles, then it maintained relatively constant sloped with a mean value of 3.765 kN until a displacement value of 5.406 mm when the fourth hinge occurs and the vault collapses. For the vault 2 the same behaviour can be observed until the third hinge occurs with a peak point load of 4.691 kN and 3.224 mm of displacement, unlike the vault 1 a further decline in value is observed until 1.855 kN, after this point the curve increases until the second peak occurs for 2.165 kN load and 5.147 mm when the mechanism occurs and the load decreases close to zero.

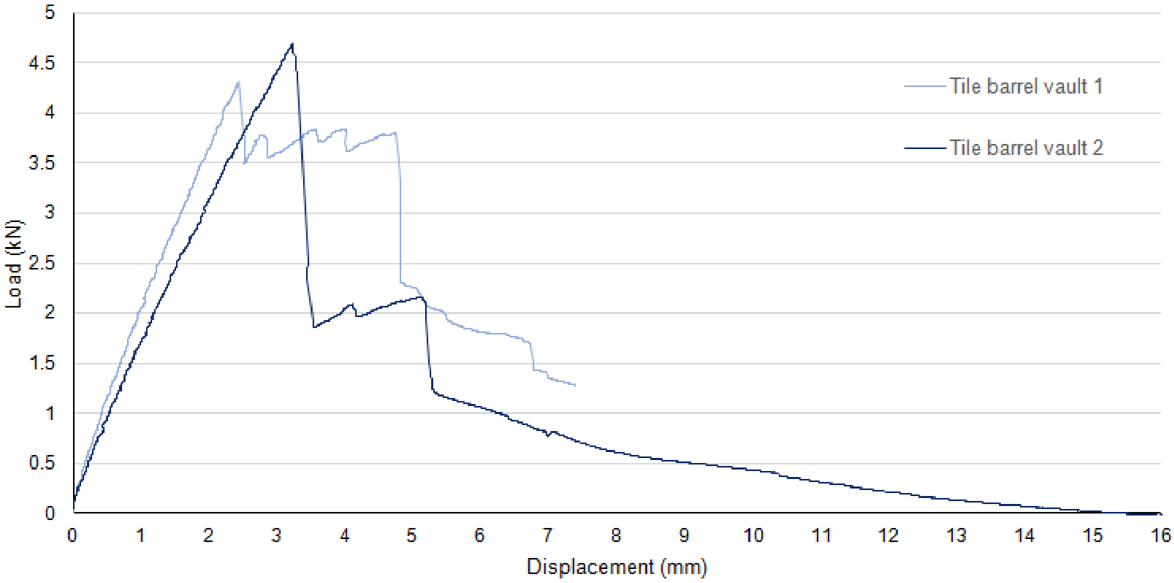


Figure 45 – Experimental results tile barrel vault (David Lopez)

Another load – displacement curve is developed considering the horizontal displacement in one support, here a similar behavior can be observed between the two vaults. For the vault 1 a linear behavior can be observed until a load of 4.319 kN, with a displacement of 0.453 mm and for the vault 2 for the 4.691 kN load value with a displacement of 0.539 mm. After these points a snap-through can be observed in both curves.

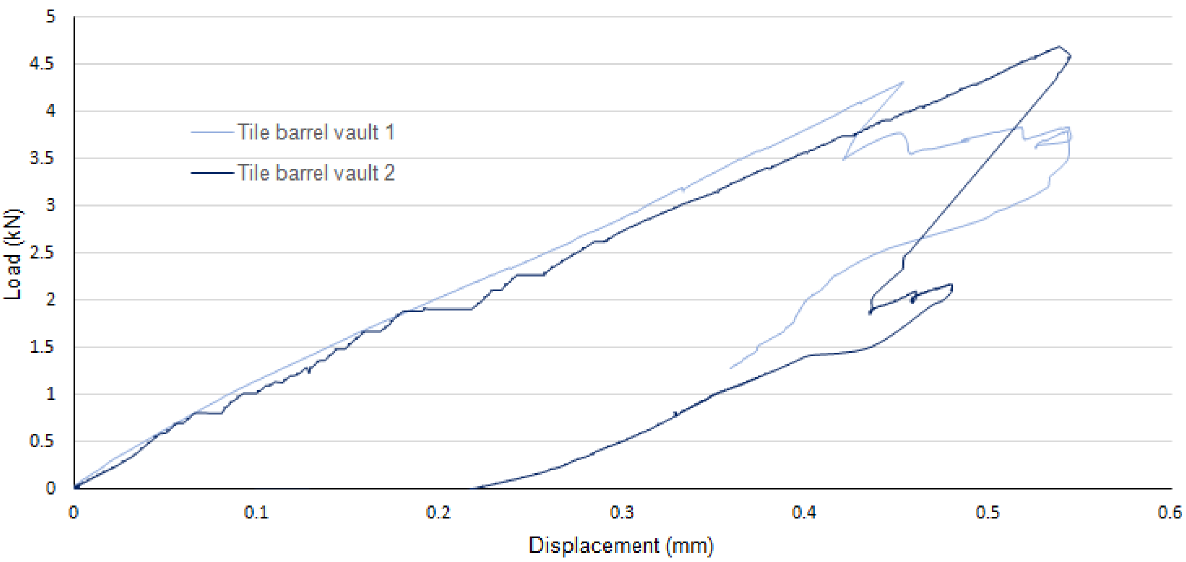


Figure 46 – Experimental results tile barrel vault (David Lopez)

For the composite barrel vault the results can be observed in Figure 47 and 48. The load – displacement curve for the vertical displacement shows in both cases a lineal behavior until a vertical load of 21.705 kN for the vault 1 and 23.856 kN for the vault 2, at this point the third hinge occurs and the nonlinear behavior of the structure starts until the peak loads, that was 52.423 kN for the vault 1 and 53.147 kN for the vault 2. For the vault 2 the curve shows a considerable decrease due the delamination between the masonry and the concrete [24], the vault 2 shown an unloading response after the peak until a displacement of 76 mm.

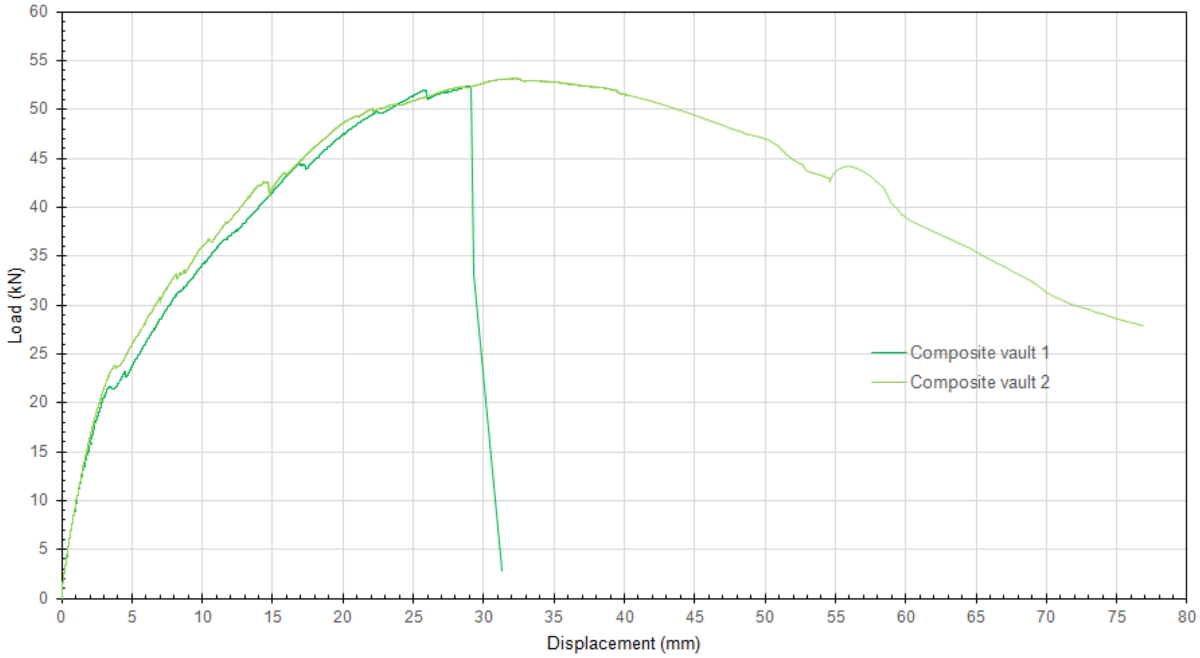


Figure 47 – Experimental results composite barrel vault (Lopez 2019)

For the load – displacement curve for horizontal displacement measured in one support, the results shown a similar behavior with a nonlinear plastic curve. For the vault 1 a linear behavior can be observed until a load of 21.705 kN, with a displacement of 0.608 mm and for the vault 2 for the 23.856 kN load value with a displacement of 0.792 mm, the displacements at the peak point were 3.909 mm and 5.962 mm.

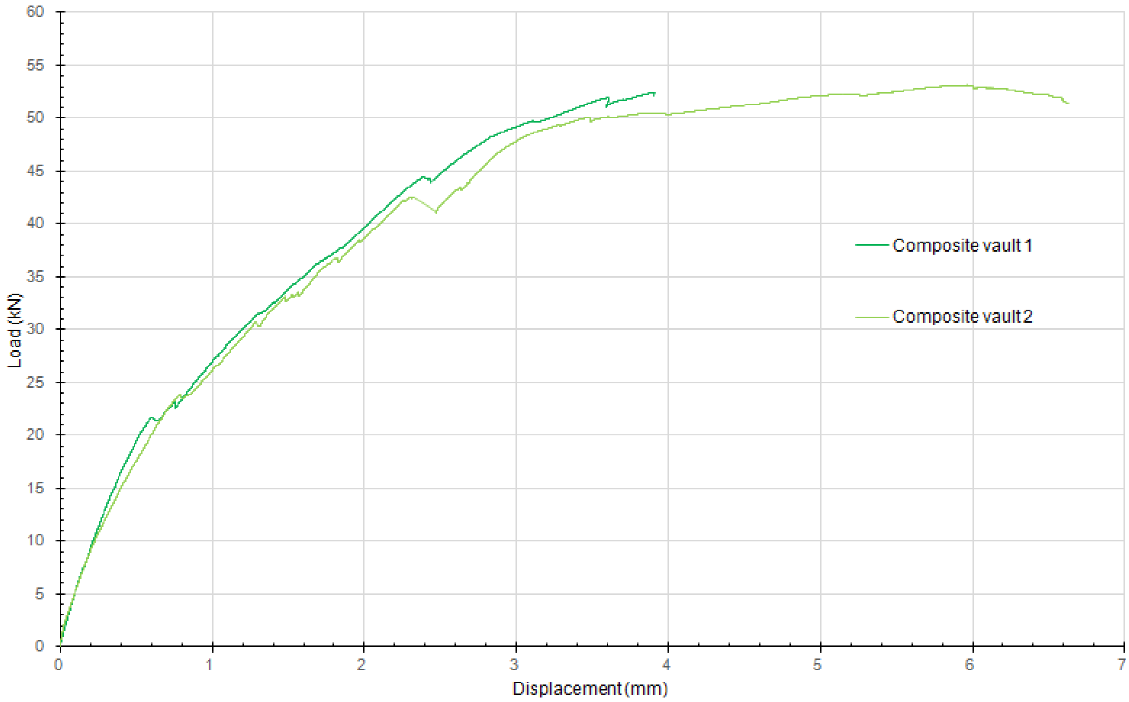


Figure 48 – Experimental results composite barrel vault (Lopez 2019)

For the sail dome, the results obtained are shown in Figure 49 and 50. The load – displacement curve for the vertical displacement shows a similar behavior for both domes, in the first part linear with peaks points of 71.358 kN with a displacement of 5.470 mm and 81.011 kN with a displacement of 6.81 mm and the second nonlinear part with peak points of 91.288 kN and 94.261 kN and displacements of 15.722 mm and 13.275 mm respectively.

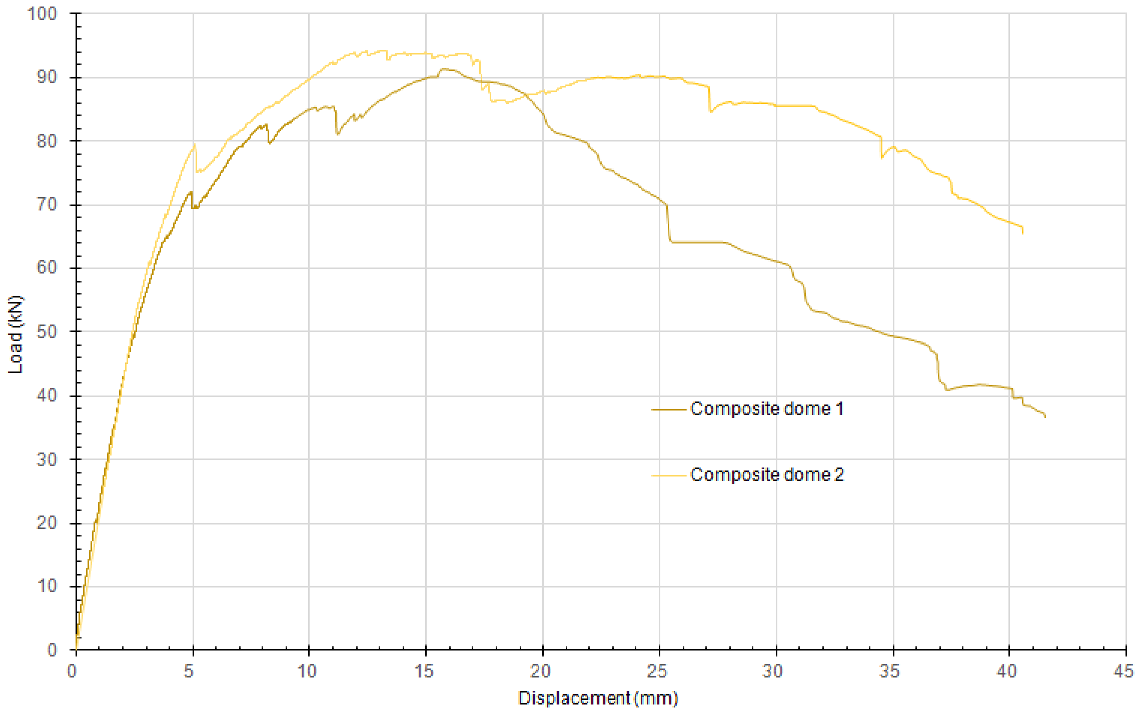


Figure 49 – Experimental results composite dome (David Lopez)

For the load – displacement curve for horizontal displacements in the support is also measured, as a difference with the others curves shown previously a high stiffness of the support system, with deformations values of 0.06 mm at 82.77 kN and 0.121 mm at 79.181 kN. After this point the stiffness decreases significantly.

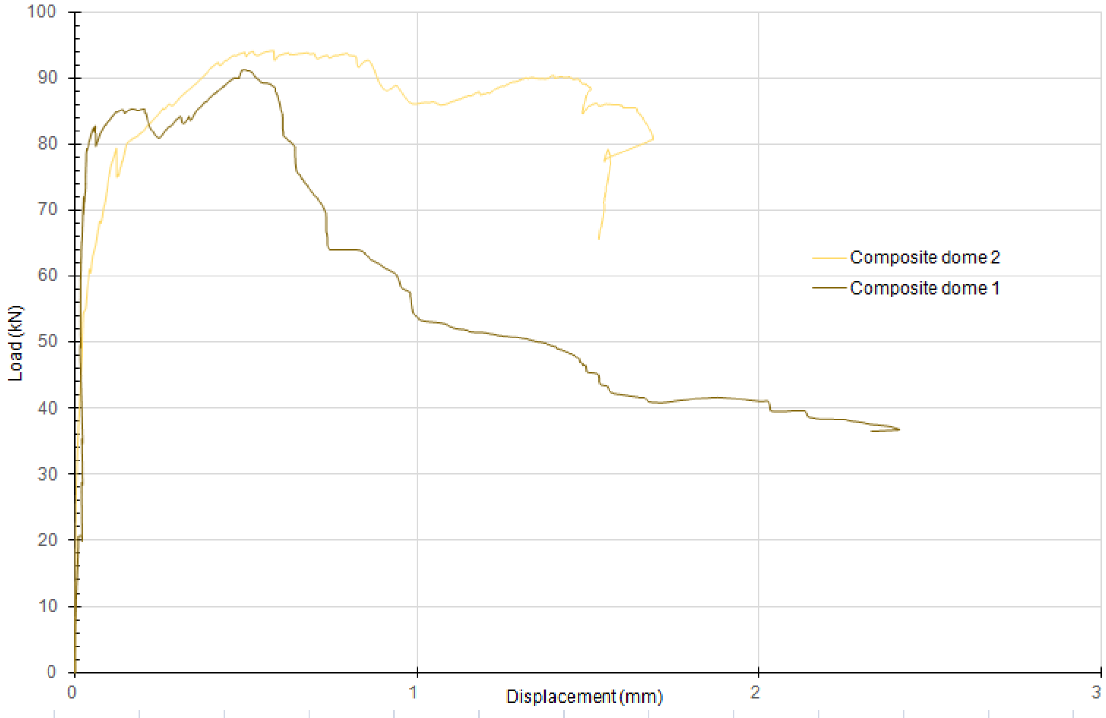


Figure 50 – Experimental results composite dome (David Lopez)

The fail mechanisms of the vaults can be observed in the following figures, the tile barrel vault can be observed in Figure 51 where the cracks appears near the support in the intrados due the traction stresses (left), other crack were observed in the extrados in the opposite side due traction stresses (right) this two points corresponds with the hinges in the vault.

Figure 51 – Cracks in tile barrel vault (David Lopez)

For the composite barrel vaults as in the tile barrel vaults the cracks are produced in the same points of the tile vault, namely in the intrados of the vault near the load platform and the extrados in the opposite side of the vault, the difference between this crack and the tile vault can be observed in Figure 52 (left) where a group of cracks are in the extrados instead a single crack in the tile (Figure 52 right), this group of crack occurs due the influence of the reinforcement [24].

Figure 52 – Cracks in Composite barrel vault extrados (left) and in the tile in the intrados (right) (David Lopez)

For the Composite dome the failure con be observed in Figure 53, where the cracks in the tile can be observed near the supports at the intrados due the tractions stresses, a group of cracks can be observed in the extrados, here the influence of the reinforcement can be observed again, a deboning between the tile and he concrete can be observed in the portion of dome between the load application point and the closest support.

Figure 53 – Cracks in Composite barrel vault extrados (David Lopez)

6. NUMERICAL SIMULATIONS

In order to simulate the behavior of vaults, appropriate models needs to be developed. For this thesis DIANA FEA software is used by considering non-linear approach and by using a constitutive model based on Total Strain Crack Model, this model is based in the idea of the stresses are described as a functions of the strains. This concept is known as hypo-elasticity when the loading and unloading behavior is along the same stress-strain path [28]

6.1 Model properties

Before performing the macro – modelling approach the geometry and the mechanical properties must be defined, in the case of geometry, for each case it is obtained from the measurements made in the vaults (Figures 36 to 40) and for the case of the supports the most reliable assumption is a pinned connection, in which the rotation is allowed.

Regarding with the mechanical properties, as mentioned in point 5.3 some values were obtained from direct test made to the constitutive materials of the vaults (see Figure 41). Other values need to be calculated from those obtained by the tests. The characteristic compressive strength of the tile courses were calculated by using the formula 3.2 of the Eurocode 6 [25].

$$f_k = K\, f_b^{\alpha}\, f_m^{\beta} \tag{2}$$

Where:
K is a constant equal to 0.55 obtained from the Table 3.3 of the Eurocode 6, considering General purpose mortar.

f_b is the normalized mean compressive strength of the tile brick, for general purpose mortar, according to the code, the maximum value to consider for general purpose mortar is 75 N/mm^2, this value is lower than the two compressive strength of the tiles tested in two directions, so the value of 75 N/mm^2 is used in the calculation of both courses.

f_m is the compressive strength of the mortar, according to the Code, for general purpose mortar the maximum value to consider is 20 N/mm^2 so both values can be used, that means every course will have a different characteristic compressive strength of the masonry.

α and β are constants, for this case the values considered were 0.7 and 0.3 respectively.

The value of f_k must be divided by 0.8 to obtain the average compressive strength.

The values obtained from the formula for each layer are shown in Table 4.

Table 4 – Average compressive strength values

First course masonry	22.126	N/mm^2
Mortar layer	6.98	N/mm^2
Second course masonry	25.291	N/mm^2

As previously mentioned, the thickness of the tiles used was 13 mm, but with 6.5 mm with grooves and the other 6.5 mm filled along the width (Figure 42). Under this consideration the thicknesses of the layers of tiles are 6.5 mm and the mortar thickness 23 mm. As a macro modeling approach, the entire tile layer it is considered as a one material, so an average compression value is calculated by multiplying the average compressive strength value by their thickness and the result divided by the entire thickness of the tile (36 mm). The value obtained is 13.021 N/mm^2. The value recommended by Professor Paulo Lourenço [26] for tensile strength of the masonry is 10% of the compressive strength, so the value considered is f_t = 1.302 N/mm^2. For the Young's modulus according to the Eurocode 6 [25] is calculated by the formula:

$$E = K_E \, f_k \tag{3}$$

Where f_k was obtained previously and the recommended value of K_E is 1000. So, the Young's modulus is equal to 13021 N/mm^2.

The Fracture energy, that is the energy required to propagate a crack in a unit of area, needs to be calculated in compression and tension also, the formulas are obtained from the Model Code 90 [27].

$$G_{fc} = 15 + 0.43 \, f_k - 0.0036 \, f_k^2 \tag{4}$$

$$G_{ft} = 0.025 \, f_t^{0.7} \tag{5}$$

Summarizing, the values considered for the masonry are shown in Table 5.

Table 5 – Tile mechanical properties

Compressive strength	13.02	N/mm^2
Tensile strength	1.30	N/mm^2
Young Modulus	13021	N/mm^2
Density	2000	kg/m^3
Poisson's ratio	0.2	
Comp. fracture energy	19.99	N/mm
Tensile fracture energy	0.03	N/mm

For the concrete, from Chapter 5, the compressive strength, the mean value obtained was 27.75 N/mm^2 and the density was 2460 kg/m^3. To transform the compressive strength from cubic samples to cylindrical samples, used in the design and modeling of concrete elements. The new value considered is 22.20 N/mm^2.

The average tensile strength of the concrete is calculated with the Model Code 90 [27]. The equation considered is:

$$f_t = 1.4\ (f_{cm}/10)^{2/3} \tag{6}$$

Where f_{cm} is the average comprevise stress of the concrete.

The young modulus is obtained from the Model Code 90 [27] with the following equation:

$$E_{ci} = E_{c0}\,\alpha_E\ (f_{cm}/10)^{1/3} \tag{7}$$

The Fracture energy in compression and tension is calculated with the formulas obtained from the Model Code 90 [27].

$$G_{fc} = 15 + 0.43\ f_k - 0.0036\ f_k^2 \tag{8}$$

$$G_{ft} = 73\ f_{cm}^{0.18} \tag{9}$$

From the equations and the tests, the following concrete properties are considered in Table 6.

Table 6 – Concrete mechanical properties

Compressive strength	22.20	N/mm^2
Tensile strength	2.30	N/mm^2
Young Modulus	28047	N/mm^2
Density	2460	kg/m^3
Poisson's ratio	0.2	
Comp. fracture energy	22.8	N/mm
Tensile fracture energy	0.13	N/mm

For Total Strain Crack Model, total strain – stress relations need to be defined for tension and compression. For the tensile behavior of the masonry the approach selected was exponential softening curve and for the compressive behavior parabolic curve, both based on fracture energy. Finally a rotating crack orientation must be considered..

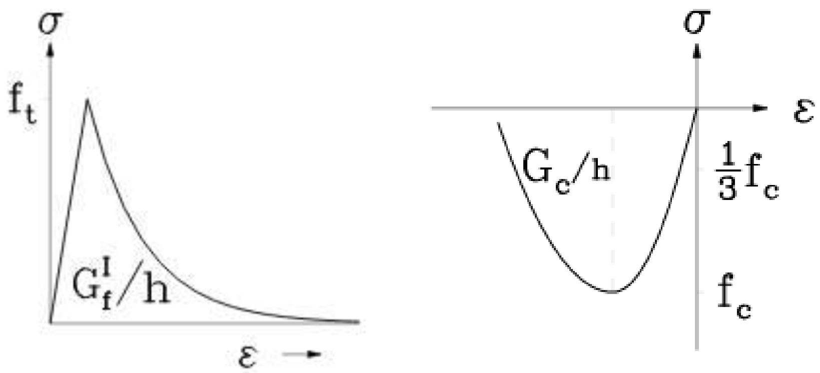


Figure 54 – Tension (left) and compression (right) strain – stress relations (DIANA FEA)

In relation with the supports conditions and the load cases, this two parameters must be included also in the model. According to the support condition, pinned support were considered in both sides allowing the rotation of the edges, this is similar to the support condition observed in Figure 43. For the loads, in addition to the self weight of the vault, a distributed load is considered to represent the external load applied during the testing; the value considered for simplicity in the load factor calculations was 1kN that was divided uniformly in the surface of 115 mm x 1000 mm which corresponds with the surface loaded in the vault prototypes. For the sail dome the loaded surface was 160 mm x 160 mm.

For the meshing quadrilateral elements were considered; the mesh generation was based in dimension (30 mm) of size, this with the purpose of obtain square elements.

The elements used for the 2D models were plane stress elements.

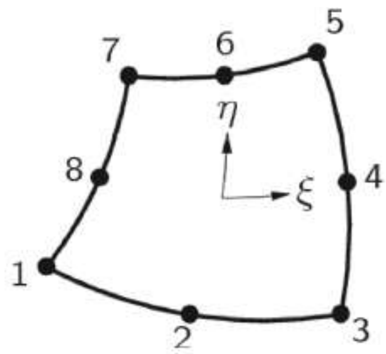


Figure 55 – Plane stress element (DIANA FEA)

For the 3D shells model curved shell elements were considered, this elements are based on isoparametric, degenerated-solid approach, that means the assumptions of normal remain straight, but not necessarily normal to the reference surface and the normal stress component in the normal direction is equal to zero.

For the Composite Timbrel – Concrete Elements, layered shell elements should be used, in these elements the full thickness is subdivided in a number of layers in which each layer has its own material properties and is numerically integrated by separated. The variables are the same of curved shells.

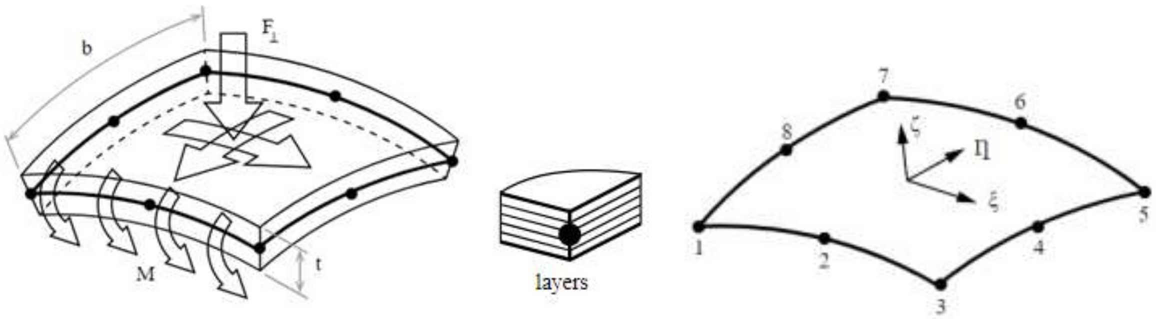


Figure 56 – Curved shell element (left) layered element (center) and integration points (right) (DIANA FEA)

Finally, for the solids, the stress situation is in three dimensions, this is useful in order the element has the same geometry than the real prototype. The number of equation needed to solve these models is higher than the others (shell), so they require more computer capacity.

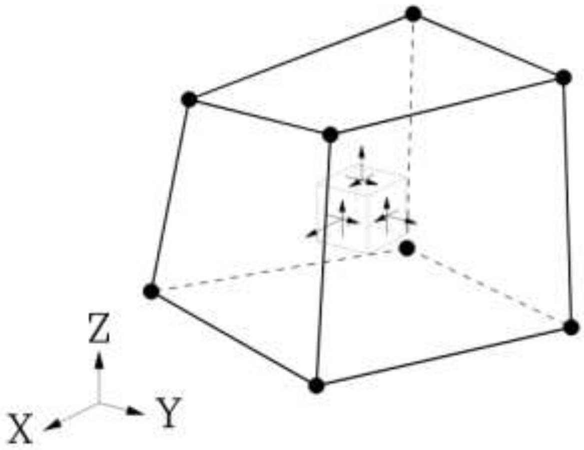


Figure 57 – Solid element (DIANA FEA)

For all the models quadratic interpolation was considered.

6.2 Tile barrel vaults

For the Tile barrel vaults, three models were developed, a 2D shell model, 3D shell model and a 3D solid model. For them the geometry was obtained directly from the measures made of the prototype and shown in Figure 36. The models created in DIANA FEA, are shown in Figures 58, 59 and 60, for the 2D model, the thickness considered was 1 meter and for the 3D shell model the thickness considered was 36 mm.

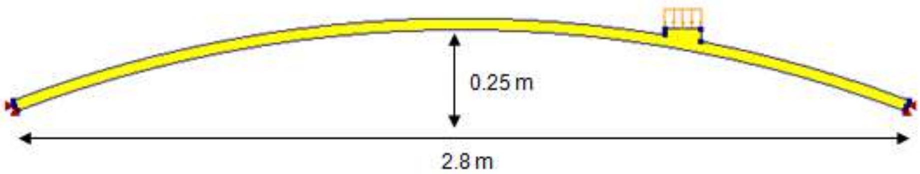


Figure 58 – Dimensions for the 2D shell model Tile vault.

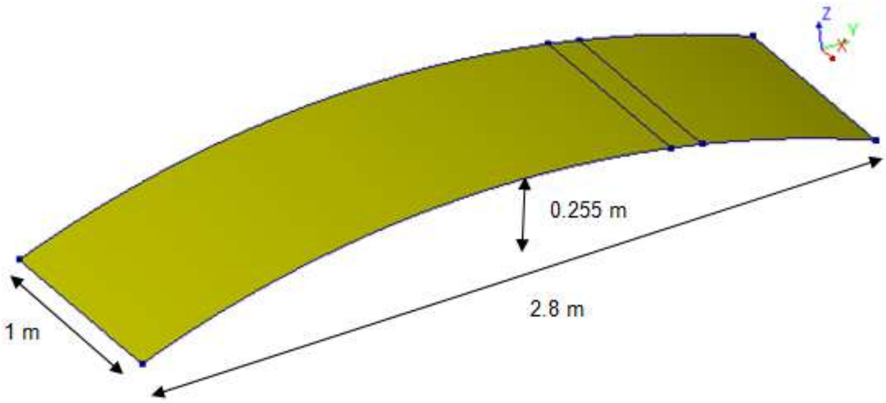


Figure 59 – Dimensions for the 3D shell model Tile vault and Composite Tile – Concrete vault.

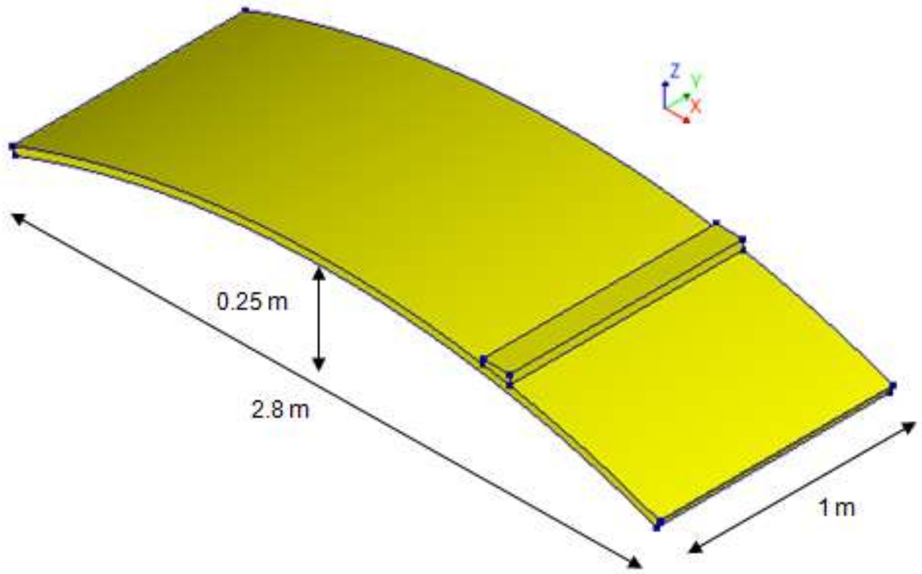


Figure 60 – Dimensions for the 3D solid model Tile vault.

The mesh of the models is shown in Figures 61, 62 and 63.

Figure 61 – Mesh of the 2D shell model Tile vault.

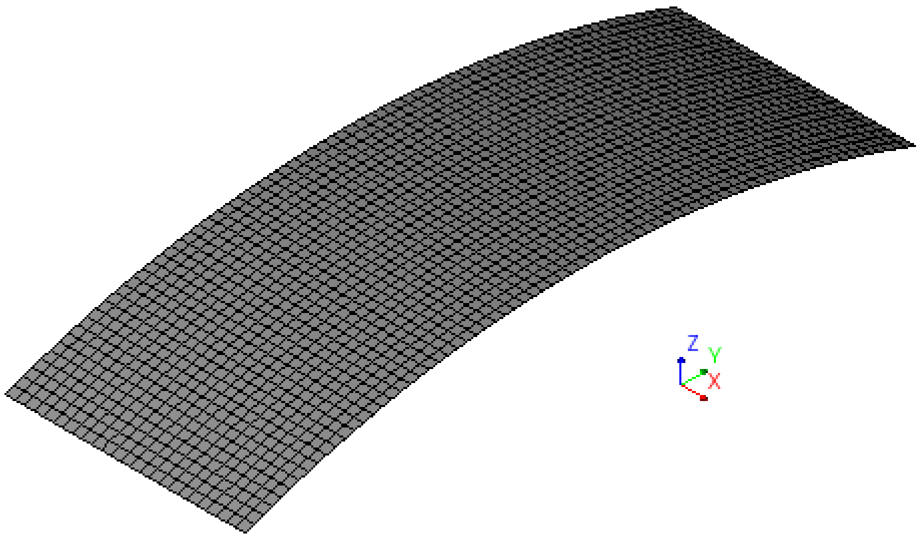


Figure 62 – Mesh of the 3D shell model Tile vault Composite Timbrel – Concrete vault.

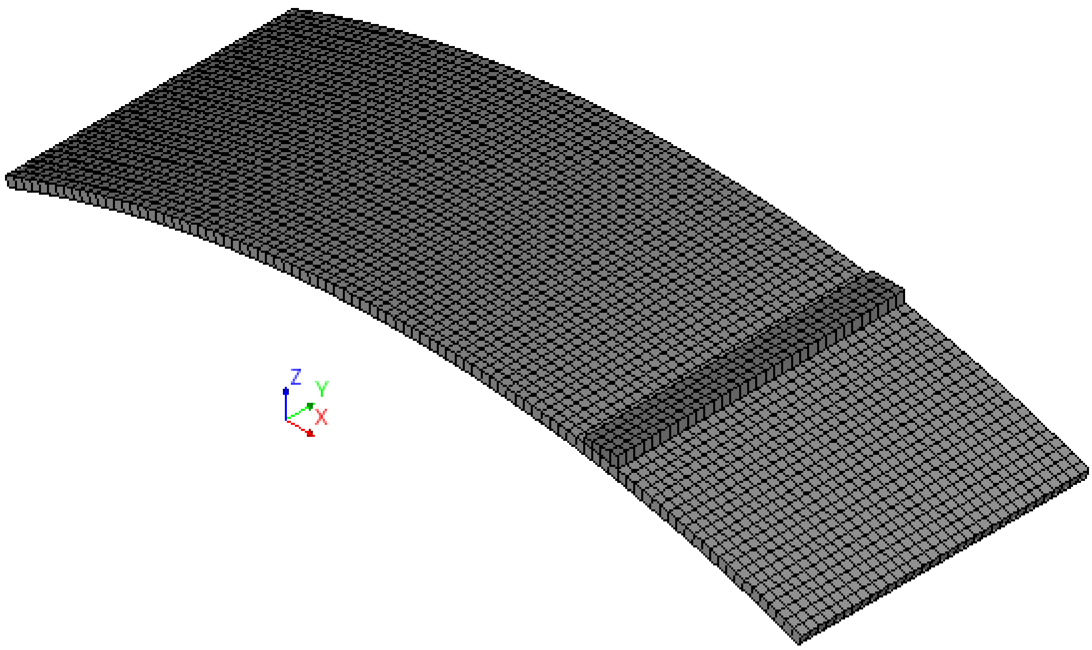


Figure 63 – Mesh of the 3D solid model Tile vault

The inclusion of the load and supports in the model is shown in Figure 64, for the Composite Timbrel – Concrete vaults 3D shell model, the load is applied in the same way. They are pinned supports and the load depends on the model to be used, for the 2D shell model the load applied was 8.695 N/mm, for the 3D shell model was 0.008695 N/mm^2 as equal for the solid model.

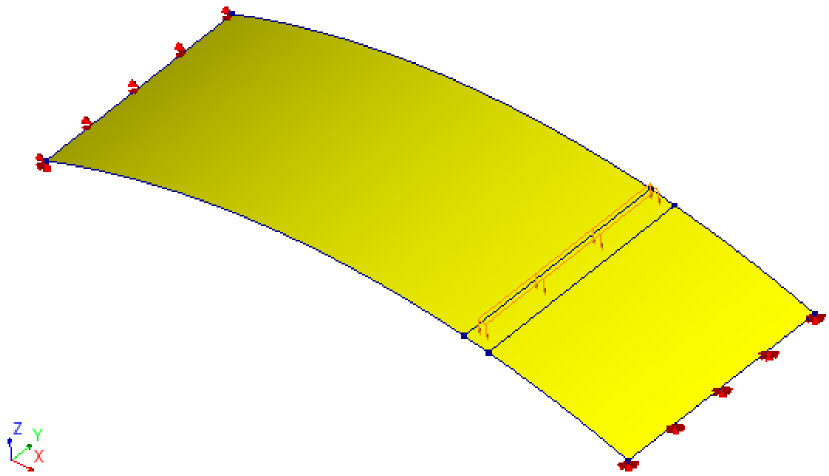


Figure 64 – Loads and supports in the 3D shell model of Tile vault and Composite Timbrel – Concrete vault.

6.3 Composite Timbrel – Concrete Barrel Vaults

For the Composite Timbrel – Concrete barrel vault, two models were developed in DIANA FEA with shell elements, one with Layered Curved Shell Elements in 3D and another with plane stress shell elements in 2D. For all the models the geometry was obtained directly from the measures made of the prototype and shown in Figure 37. For the 3D model, the full thickness considered was 86 mm and the factors included in DIANA for consider the tile and concrete were 0.418 and 0.582 and for the 2D model the value considered was 1 meter. The dimension of this vault is shown in Figures 65 and 66.

For the composite models the reinforcement needs also to be included, for the 3D shell model is included as a new shape with reinforcement properties where the diameters of the bars and their spacing are defined, this shape was displaced from the axis of the vault 13.82 mm, and for the 2D model as a line which represents the sum of the areas of all the bars in the width of the vault, at the beginning tried to include reinforcement as a layer but the results were not satisfactory.

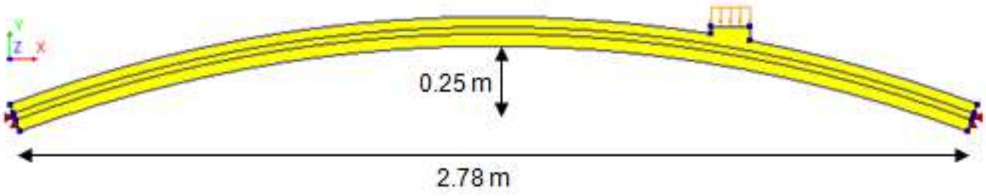


Figure 65 – Load application and supports in the 2D shell model. The reinforcement can also be observed

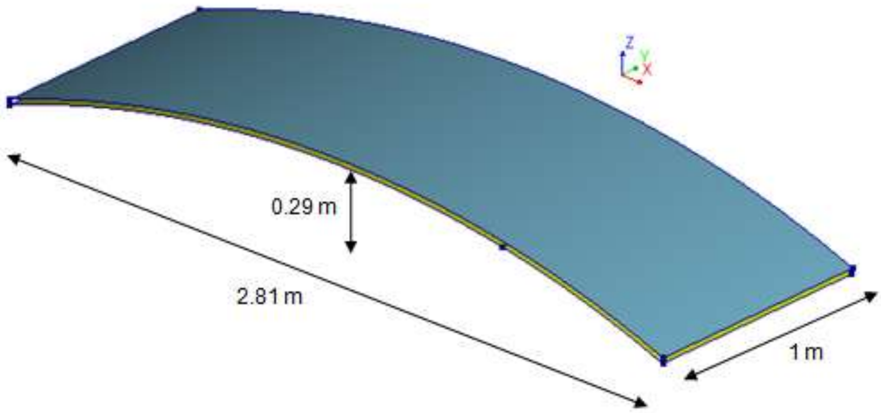


Figure 66 – 3D layered shell model, the blue shape corresponds to the reinforcement.

The mesh of the models is shown in Figures 67and 68.

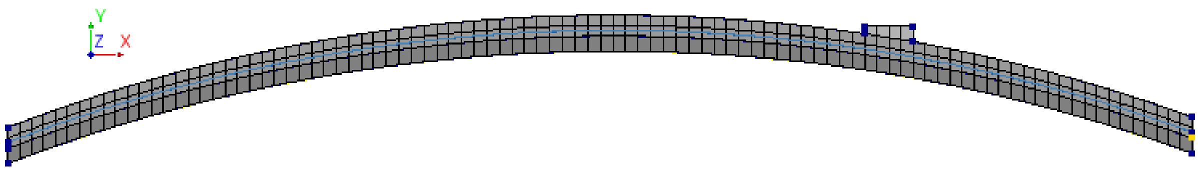


Figure 67 – Mesh of the 2D shell model Composite Tile – Concrete vault.

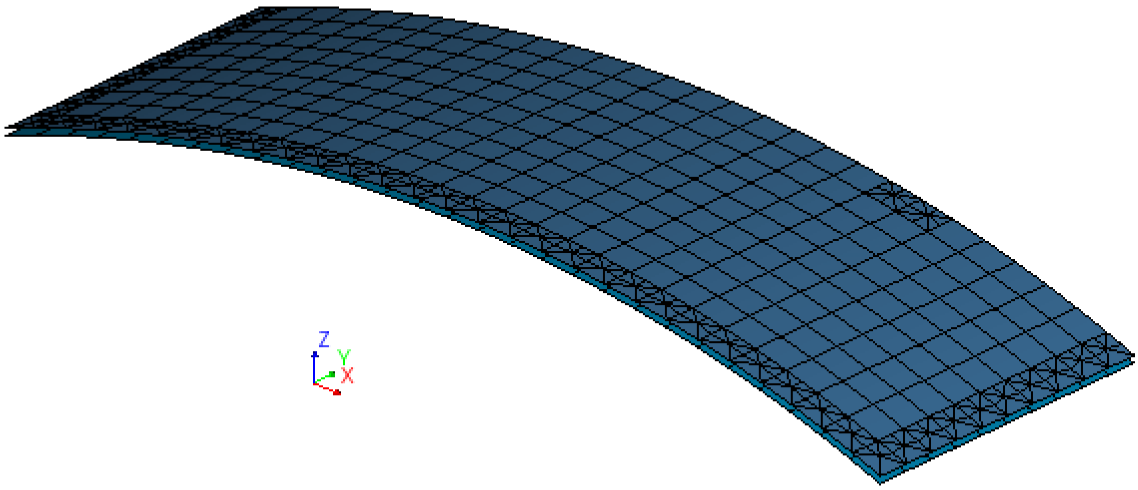


Figure 68 – Mesh of the 3D shell model Composite Tile – Concrete vault.

6.4 Composite Timbrel – Concrete Sail Dome.

For the Composite Timbrel – Concrete Sail Dome, only one model were developed in DIANA FEA with Layered Curved Shell Elements in 3D. The dimensions were obtained from the prototypes (Figures 38, 39 and 40) and are shown in Figure 69.

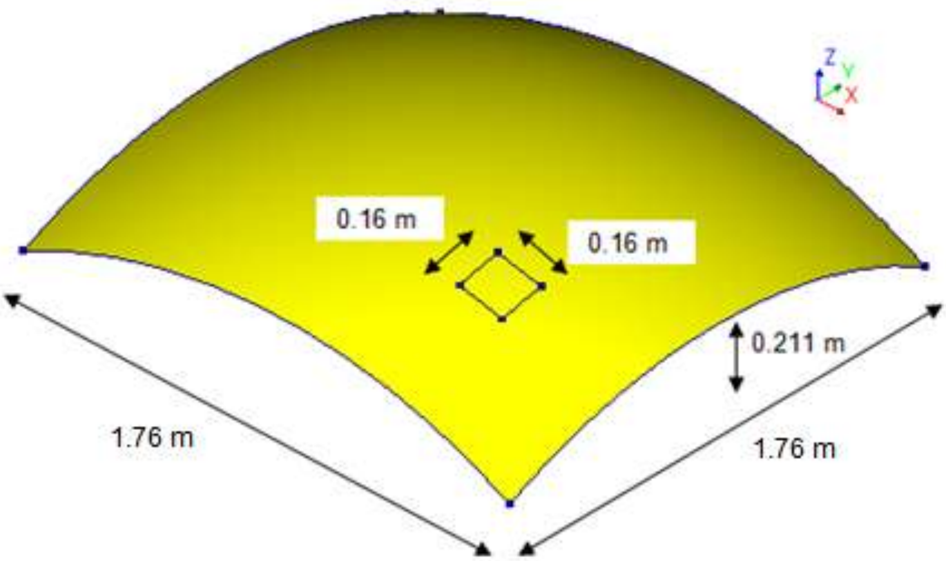


Figure 69 – Dimensions of Shell model Tile vault and Composite Tile – Concrete Sail Dome.

As in the Composite Tile – Concrete Barrel vault 3D shell model, a new shape with reinforcement properties where the diameters of the bars and their spacing are defined, the distance between both shapes is measured between the axis of the vault and the axis of the reinforcement.

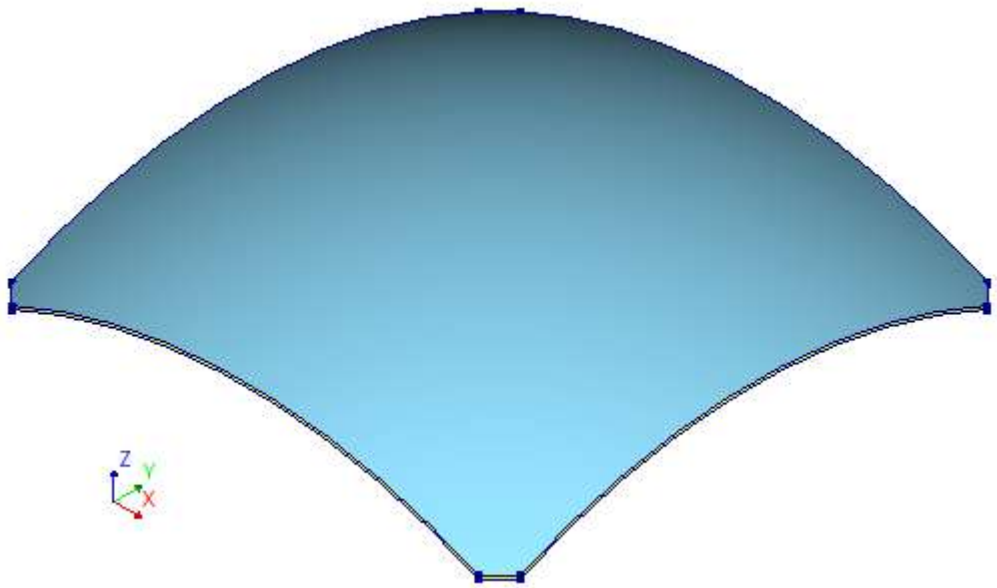


Figure 70 - Shell model Tile vault and Composite Tile – Concrete Sail Dome.

The mesh of the model is shown in Figure 71

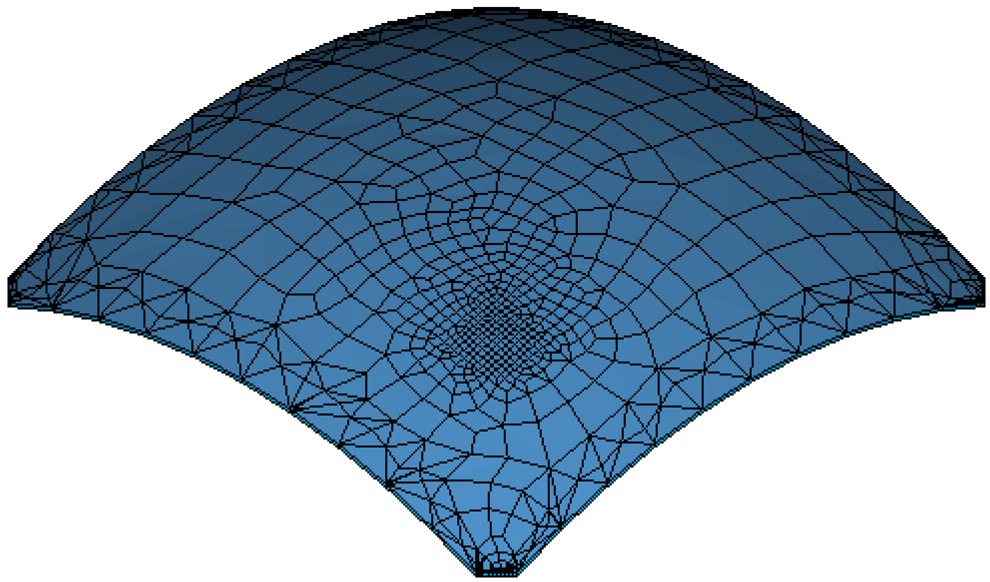


Figure 71 - Shell model Tile vault and Composite Tile – Concrete Sail Dome.

6.5 Results

From the models developed to perform the numerical analysis, several load – displacement curves have been obtained. The first model considered the mechanical properties calculated in the Point 6.1. From that starting point the curves were plotted, and the results obtained they were not similar to those obtained experimentally. In order to obtain a better fit between the experimental and Finite Element Model curves some mechanical properties values were changed. The mechanical properties changed cannot be those obtained directly from laboratory tests and that were described in Point 5.3.

For the three models performed for the Tile vault, the collapse mechanism is shown, as a reference the principal strains will presented. For the 2D shell model, only 2 points are shown in Figure 72, the first is when the external load applied was 1 kN and when the third hinge appears, for this case only 1 hinge can be found and its located near the load application point in the intrados.

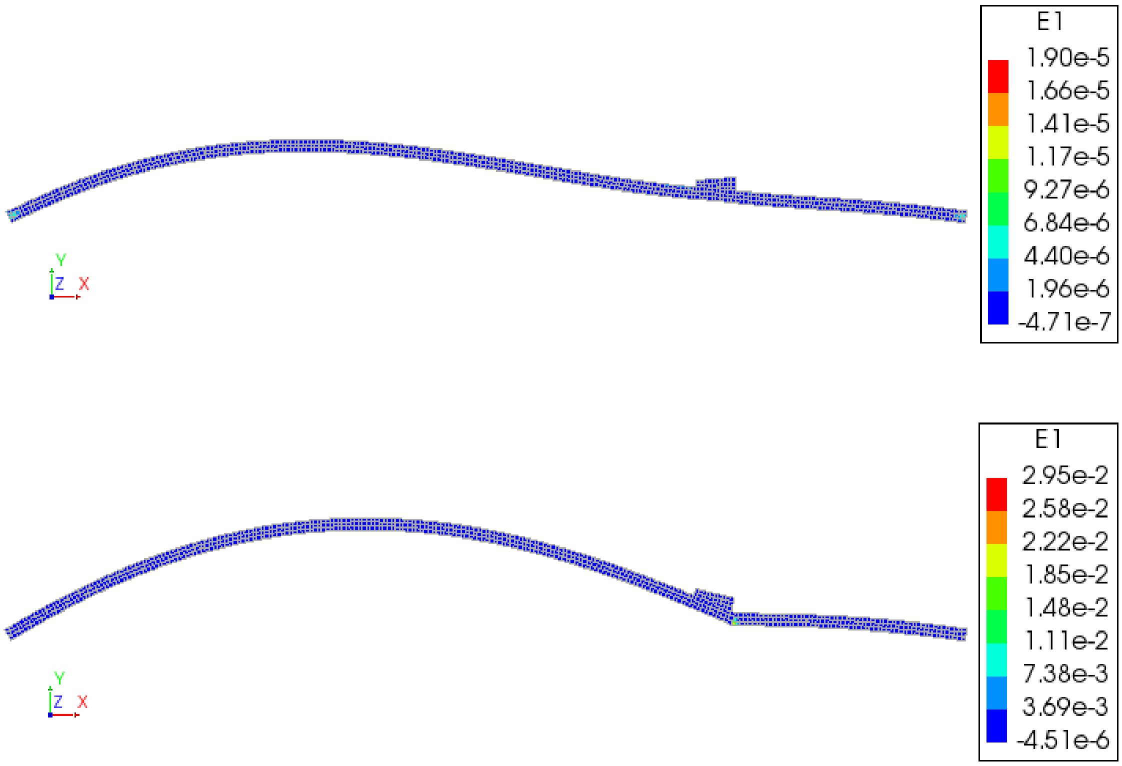
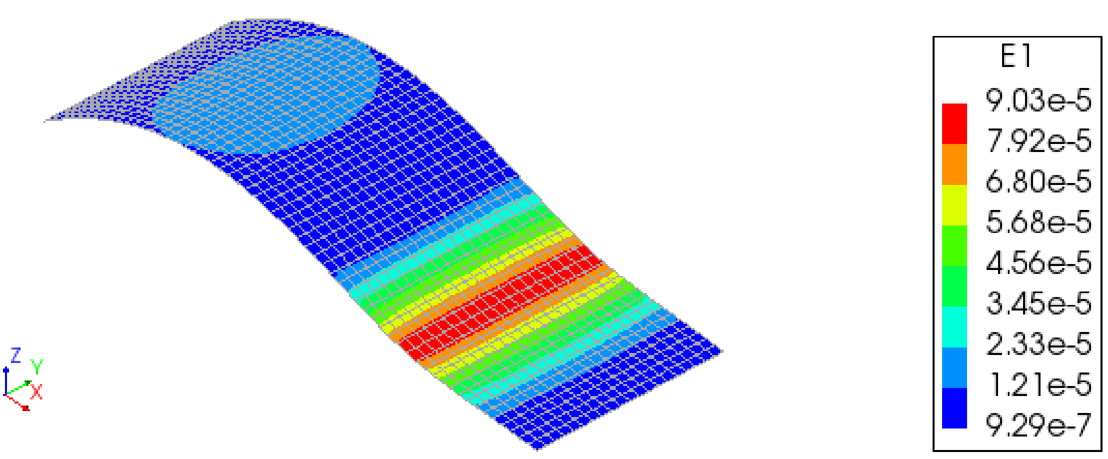


Figure 72 – Principal strains 2D Shell model of Tile vault.

For the 3D shell, the collapse mechanism in observed in Figure 73, the collapse occurs in the load application point first (3rd hinge) and in the opposite side of the vault occurs the collapse of the structure (4rd hinge).



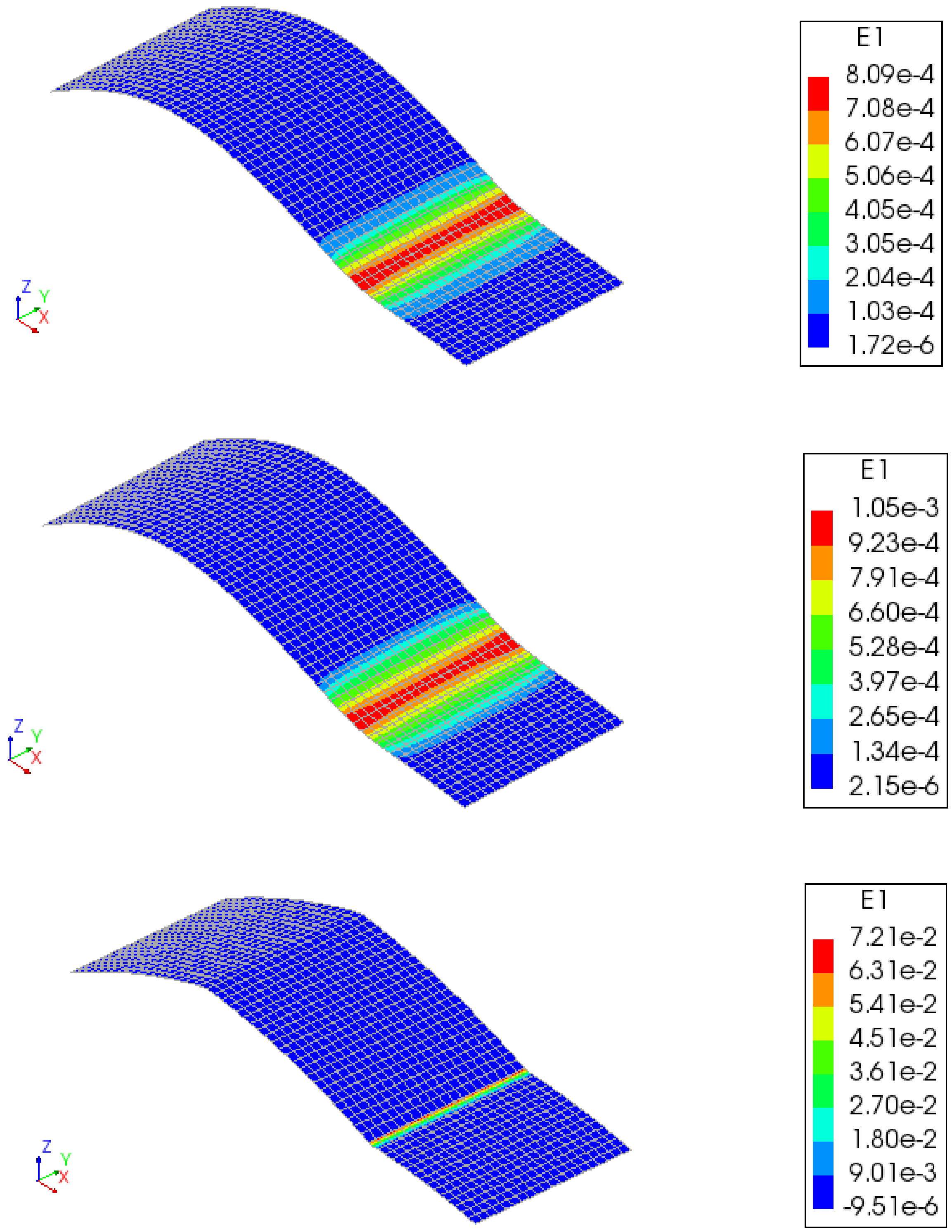


Figure 73 – Principal strains at different loads (2kN, 4 kN, 4.77 kN and after collapse) of 3D shell model

For the 3D solid model the Collapse mechanism id observed in Figure74, in it is easier to observe that the cracks. For the 3rd hinge, the cracks appear in the intrados of the vault and for the 4th hinge the cracks appears in the extrados. This behaviour of the vault is similar that observed for the Timbrel vault in Figure 51.

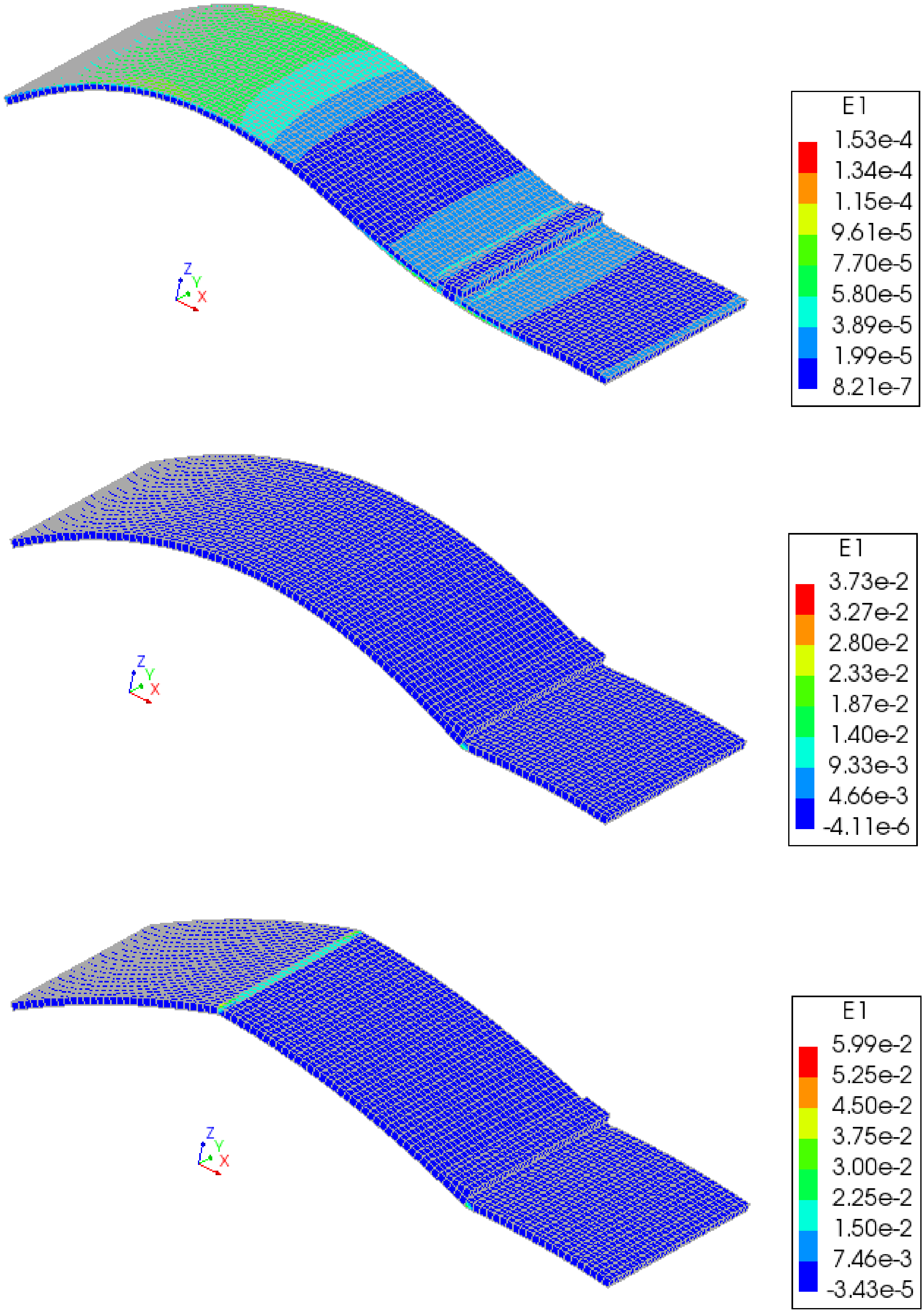


Figure 74 – Principal strains at different loads (2kN, after 3rd hinge and after collapse) of 3D solid model.

The Load – displacement curve for the Timbrel vault is shown in the Figure 75, where in blue and light blue the experimental values are shown, in red is the 3D shell, in orange the 3D solid and finally in magenta the 2D shell. For the 3D shell the two peaks due the two hinges is clearly visible, the maximum peak occurs at 4.77 kN at 5.1 mm of displacement, after that point the vault stops resisting load until a load factor of 1.864 kN. When the vault starts resisting loads again until a second peak at 2.66 kN, that is when the 4th hinge occurs and the vault collapse. For the 3D solid model, the behavior of the curve is very similar to the 3D shell model with different peak values; in this case the values are 3.693 kN and 2.268 kN. As previously mentioned, for the 2D shell is represented only with one peak value at 4.00 kN. Except for the 3D shell model, the load strength values were lower than those obtained in the experiments; also, the three numerical models present a similar stiffness, except the 3D shell near the first peak. The 3D solid model and the 3D shell model presents similar behavior with two peaks with experimental data of the Tile barrel vault 2, but for the models a higher displacement was obtained.

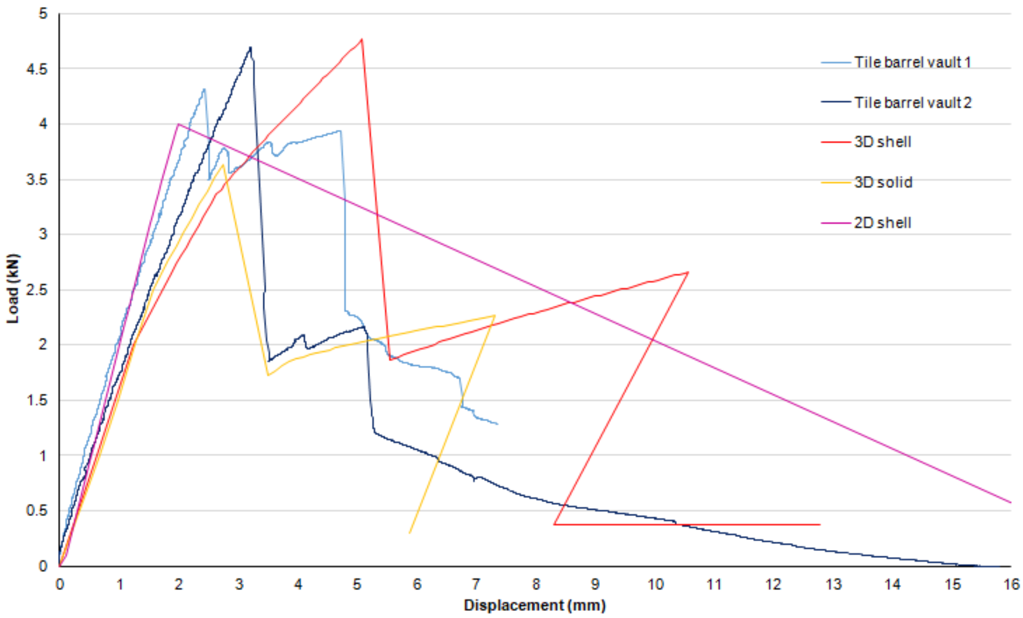


Figure 75 – Load – displacement curve for Timbrel vault.

For the Tile vault, no change in the mechanical properties from those indicated in Table 5 was necessary.

In the case of the Composite Timbrel – Concrete Vault, the collapse mechanism can be observed from the principal strains. In the 2D model, can be observed that the 3rd hinge appears in the intrados near the load application point and the 4th hinge appears in the opposite side of the vault but in the extrados, but it is looks more widespread due the effect of the reinforcement. This behaviour is similar to the Composite Timbrel – Concrete barrel vault shown in Figure 52.

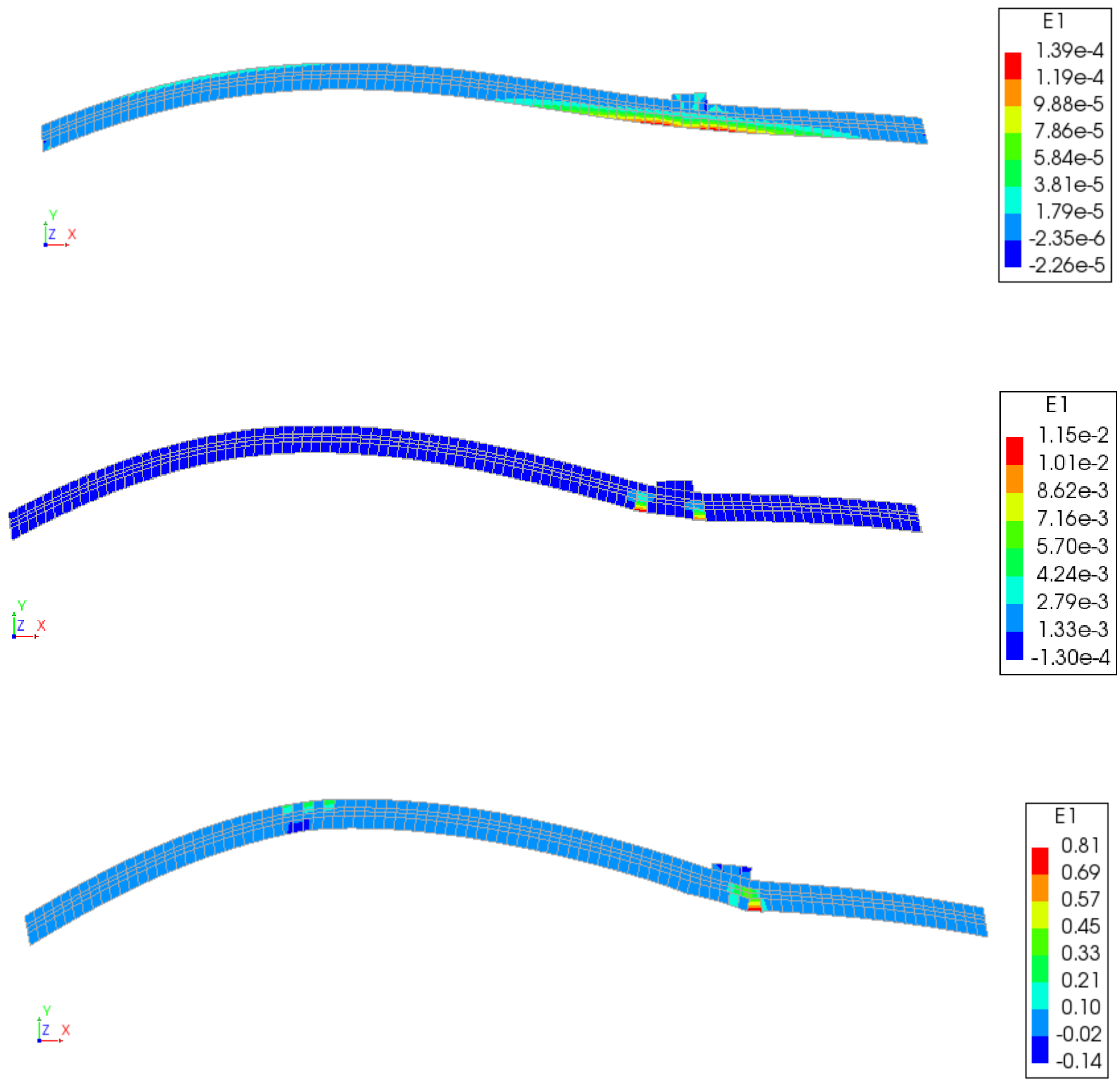
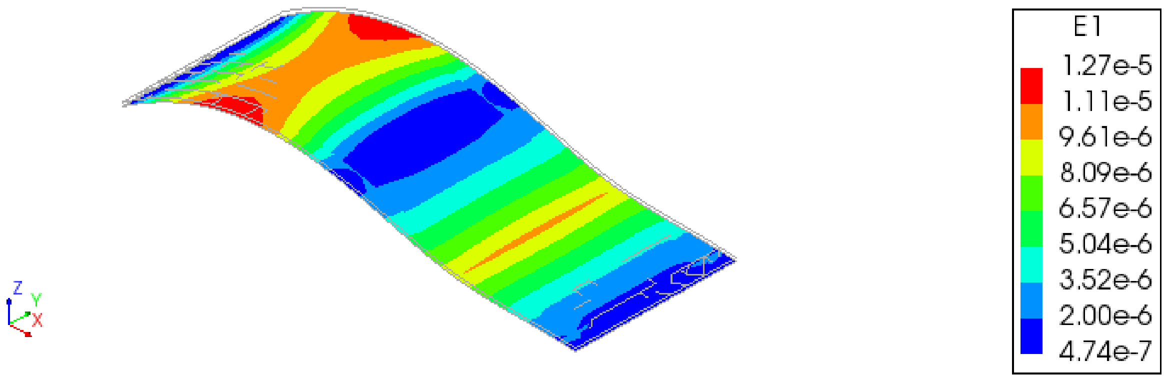


Figure 76 – Principal strains at different loads (10 kN, after 3rd hinge and after collapse) for 2D shell model.



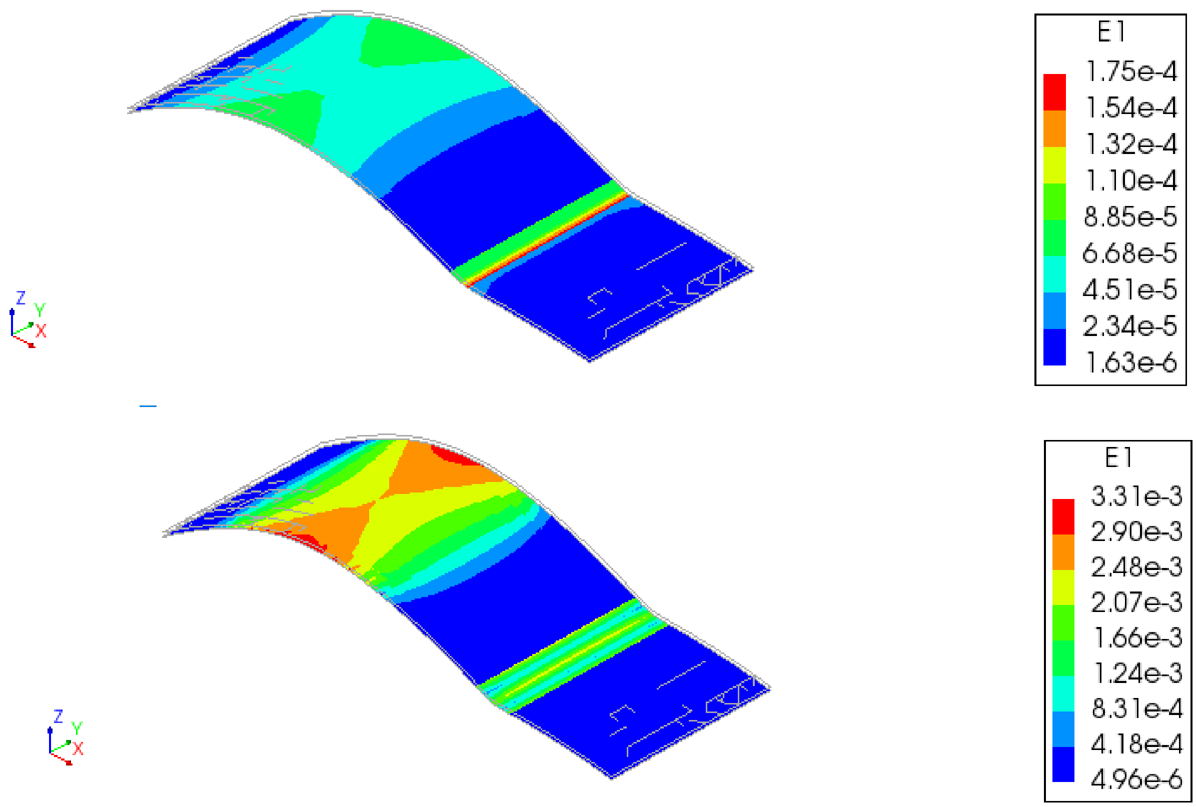


Figure 77 – Principal strains at different loads (10 kN, after 3rd hinge and after collapse) for 3D layered shell model.

From the analysis of the both models, each load – vertical displacement curve is obtained, this curves are compared with the experimental ones (Figure 78), the blue curve corresponds with the 3D shell model and the red curve the 2D shell model. For the 3D shell model, the strength capacity of the vault is 52.967 kN and for the 2D shell model is 50.989 kN, these values are close to the experimental (52.423 kN and 53.147 kN). The difference observed is the stiffness of the model, which is higher than the observed in the experiments, one of the reasons of this difference could be de displacement observed in the support. For that reason the 2D shell model has a spring in one of the edge in which the displacement was measured and the stiffness of the spring is calibrated with the load – horizontal displacement curve (Figure 79).

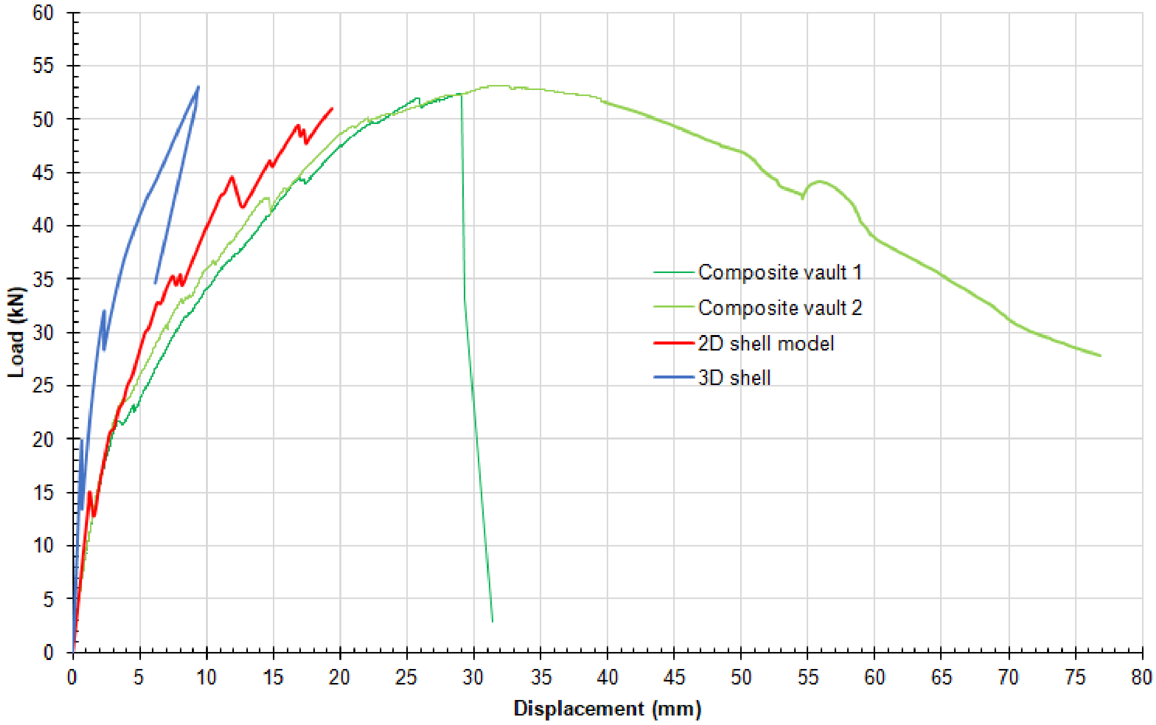


Figure 78 – Load – vertical displacement curve for Composite Timbrel – Concrete barrel vault.

From Figure 79, it can be observed that a similar slope between the experimental and numerical is obtained for the linear part of the curve, about a load factor of 23 kN, so the stiffness of the spring estimated was 45000 N/mm. From this calibration of the 2D Model the red curve in Figure 79 is obtained. It can be observed that has a better correlation than the 3D shell model.

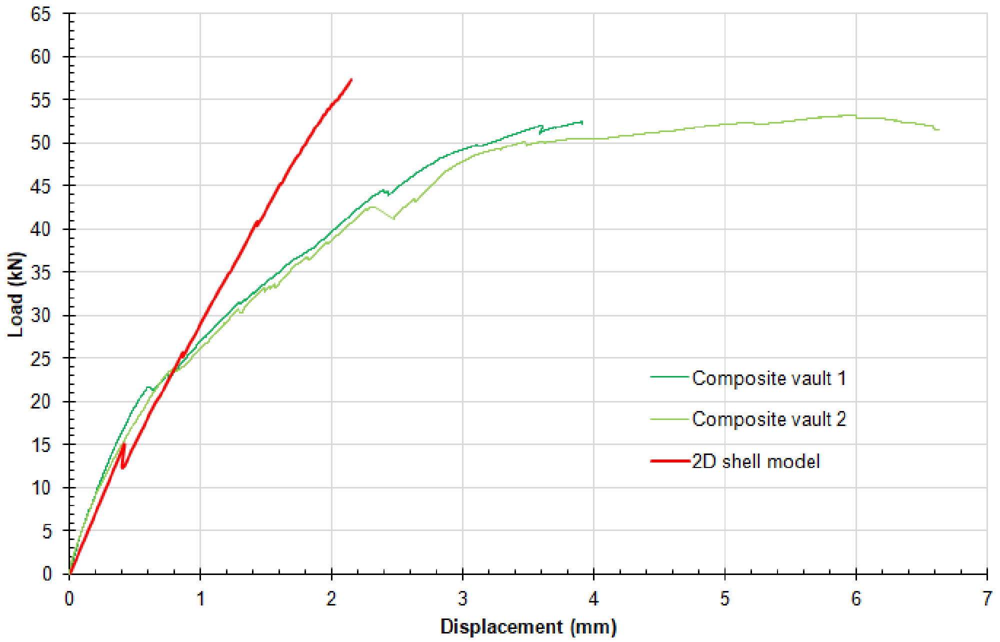


Figure 79 - Load – horizontal displacement curve for Composite Timbrel – Concrete barrel vault.

Finally, for the Composite Timbrel – Concrete Sail dome, the principal strains in the peak point obtained presents some cracks coming from the load application point to the perpendicular edges, something similar can be observed in the Figure 53 except that in the model the cracks for the negative moment in the top of the dome cannot be found in the model, because the model only analyzed until the first peak apparition.

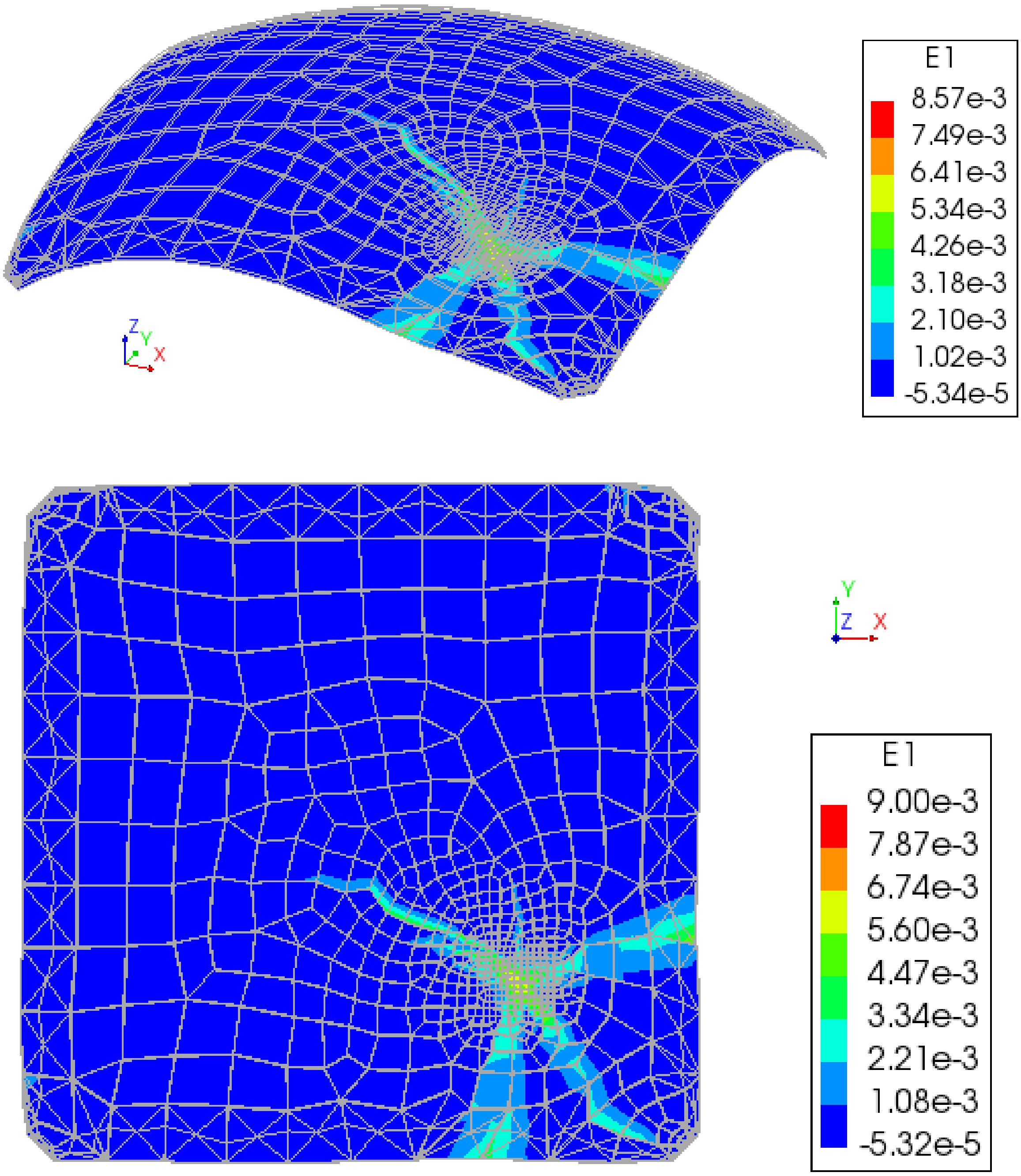


Figure 80 – Principal strains of the Composite Timbrel – Concrete sail dome in the peak point.

From the load – displacement of the sail dome, the two prototypes tested curves are presented in brown and beige. The result of the numerical analysis of the sail dome is shown in red. It can be observed that the Finite element model has more stiffness than the experimental results in the linear behaviour of the dome, this could happen because of the support conditions of the dome are not well represented in the model, don't allowing displacements in the supports. The peak value obtained was 85.18 kN which is much lower than the maximum strength of the sail dome (94.26 kN) but is closer to the elastic peak point which is 81.011 kN.

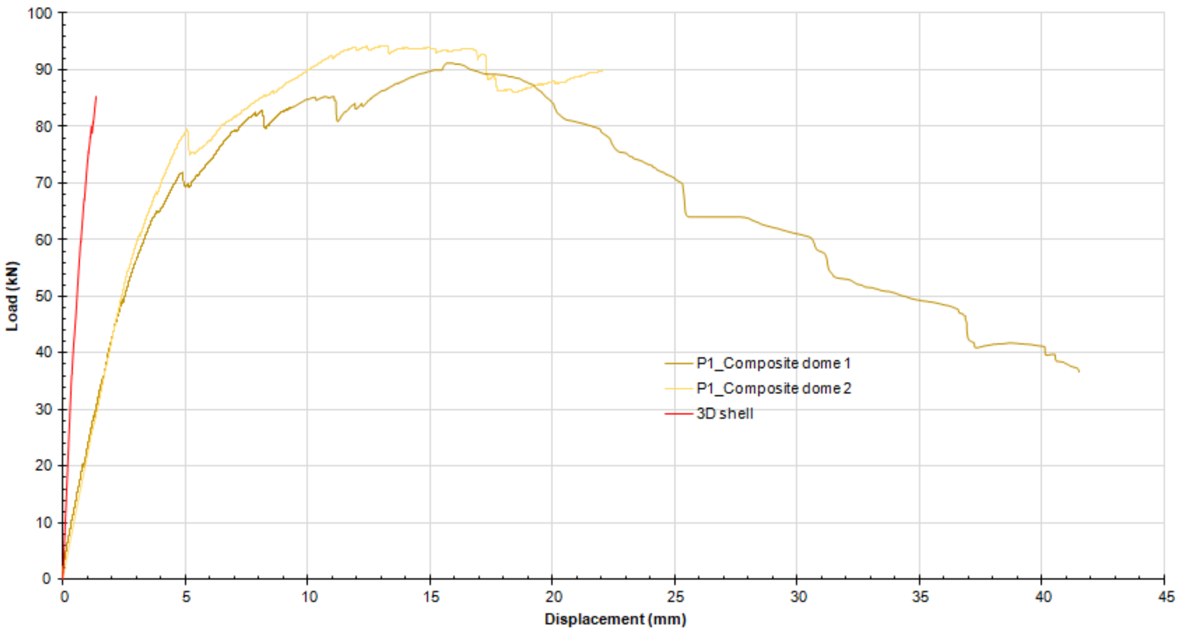


Figure 81 - Load – horizontal displacement curve for Composite Timbrel – Concrete sail dome.

Finally, In the case of the Composite Timbrel – Concrete vaults, the mechanical properties were changed in order to obtain a better correlation with the curves obtained from the tested prototypes. The values considered are shown in Table 7. Can be observed that the values of the Young's modulus and the tensile strength were reduced from the theoretical values given in the Model Code.

Table 7 –New Mechanical properties considered for the concrete

Compressive strength	22.20	N/mm^2
Tensile strength	0.90	N/mm^2
Young Modulus	20.000	N/mm^2
Density	2460	kg/m^3
Poisson's ratio	0.2	
Comp. fracture energy	22.8	N/mm
Tensile fracture energy	0.13	N/mm

7. CONCLUSIONS

From the results obtained in the force – displacement curves, both for the obtained from the laboratory tests performed by David Lopez in the UPC Facilities and the obtained from the numerical analysis in Finite Elements, it can be concluded that the models may represent the behavior of the timbrel vaults under analysis. The differences obtained between both curves can be explained mainly because of the uncertainties in the mechanical properties estimated and not tested, the assumptions considered, like the influence of the grooves in the calculation of the thickness of the tile layer and consequently the compressive strength and the support conditions, that was specially analyzed for the composite Timbrel – Concrete barrel vault who has the greatest displacements. Ideally the deflections of the supports should be avoided during the test, or at least measured in more point, because the distribution of thrust forces is not uniform in the edges.

Following the idea of the previous paragraph, the uncertainties of the materials is a constant in the development of numerical models of masonry and timbrel vaults, so what can work for a vault, will not necessarily work for another experimental study. For the support conditions determine the stiffness when the force – horizontal displacement curve is not linear it is a challenge due the non – linear spring needs to represent all the deformations due to the support system and their connections (steel shapes deformation, loss of pre-tension of the bolts, etc. Independent of the slope of the curves, the behavior of the timbrel and composite vaults can be described by the numerical analysis until the collapse; also the peak point values are similar.

In the case of the Composite Timbrel – Concrete Sail Dome, the curve obtained from the numerical analysis has not a good correlation with the experimental data, possibly due to an insufficient modeling of the supports and should be revised.

In relation of the capacity of the Timbrel vaults and the Composite Timbrel – Concrete vaults, the results obtained demonstrates the high resistance of the timbrel vaults, for example the idea expressed by Guastavino that one person can walk above the vault as soon as this is finished makes sense because the third hinge occurs at 4.54 kN. Even more interesting is the idea of the Composite Timbrel – Concrete vault which has its resistance increased more than 10 times only using a small concrete thickness and small reinforcement diameter.

Finally, the use of macro modeling approach in Finite elements is a powerful tool, especially for 3D vaults and domes, especially for complex geometries because the stress and strain distribution can be obtained in every point and the thickness can be increased in places which tension stresses occur. Nevertheless for barrel vaults classic approaches are still very useful and powerful analysis tool, especially the new procedures for Composite Timbrel – Concrete like ELARM.

8. REFERENCES

[1] Heyman, J. Estructuras de Fábrica Volumen II. ed. Instituto Juan de Herrera. Madrid: Herrera 2015.

[2] Cardellach, F. Filosofía de Las Estructuras. Barcelona: Editores Técnicos asociados. 1970.

[3] Choisy, A. L'Art de Batir chez les Romains. Paris. Ancien Eleve de L'École Polytechnique, Ingénieur Des Ponts et Chaussees. 1873

[4] Choisy, A. L'Art de Batir chez les Byzantines. Paris. Ancien Eleve de L'École Polytechnique, Ingénieur Des Ponts et Chaussees. 1883

[5] Bassegoda, B. La bóveda Catalana. Barcelona. Numerario de la Escuela Superior de Arquitectura y de la Real Academia de Ciencias y Artes.1947

[6] Huerta, S. The Mechanics of Timbrel Vaults: A Historical Outline. 2003.

[7] United States Patent Office. Patent Nº 464,562. New York. 1891

[8] United States Patent Office. Patent Nº 468,871. New York. 1892

[9] United States Patent Office. Patent Nº 947,177. New York. 1910.

[10] United States Patent Office. Patent Nº 1,052,142. New York. 1913.

[11] Rubió, S. Cálculo Funicular del Hormigón Armado. Ediciones Gili. 1952

[12] López, D., Van Mele, T., Block, P. The combination of tile vaults with reinforcement and concrete, International Journal of Architectural Heritage. 2018.

[13] Moya, L. Bovedas tabicadas. Ministerio de la Gobernación. Dirección General de Arquitectura. 1947.

[14] Gulli, R. La huella de la construcción tabicada en la arquitectura de Le Corbusier. Las bóvedas de Guastavino en América. 1999.

[15] López, D. Van Mele, T. Block, P. Dieste, González Zuleta and Sánchez del Río: Three approaches to reinforced-brick shell structures, Taylor & Francis Group, London, 2016.

[16] García, J. González, M., Losada, J. C. Tile vault architecture and construction around Eduardo Sacriste. Informes de la Construcción 2012; Vol. 64, 525, 35-50.

[17] López, D. Van Mele, T. Block, P. Tile Vaulting in the 21st Century. Informes de la Construcción 2016. Vol 68.

[18] Heyman J. Estructuras de Fábrica Volumen I. Instituto Juan de Herrera. Madrid: 2015,

[19] Saliklis, E. P., Kurtz, S. J., Furnbach, S. V. Finite Element Modeling of Guastavino Tiled Arches. Structural studies, repairs, and maintenance of heritage architecture VIII. 2003. 257–66

[20] Atamturktur S., Boothby T. E., The Development of Finite-Element Models and the Horizontal Thrust of Guastavino Domes. The Journal of Preservation Technology Vol. XXXVIIII, No.4 2007

[21] Rei, J. C. M., Gago A. S., Timbrel Vault: a Traditional Constructive Technique. 3rd International Conference on Protection of Historical Constructions. 2017.

[22] López, D., Domènech M., Tile Vaults. Structural Analysis and Experimentation. Guastavino Biennal 2015.

[23] Dhaenens, J. Exploration of the Abilities of Free-Handed Masonry Vaults Jerom Dhaenens."

[24] López, D., Roca, P., Liewc, A., Van Melec T., Block P.Tile Vaults as Integrated Formwork for Reinforced Concrete: Construction, Experimental Testing and a Method for the Design and Analysis of Two-Dimensional Structures. Engineering Structures 188 (March): 233–48. 2019.

[25] EC6 Eurocode 6: Design of masonry structures. EN 1996-1-1: 2005.;

[26] Lourenço, P. Modelling of masonry and homogenization, Universidade do Minho. 2018.

[27] Comite Euro – International du Beton, CEB – FIP Model Code 1990 Design Code.

[28] DIANA User's Manual Release 10.3. 2019.

.